I.I.T. Kharagpur

(CS60014) Quantum Computing and Quantum Information Processing

Marks:100 (Time: 2 hours 2-4 p.m. 21/02/2004)

Department of Computer Science and Engineering

Answer as many questions as you can.

Give rigorous and complete arguments written neatly.

1. Show that the Fredkin gate is universal for classical computation. Develop the unitary matrix for the quantum Toffoli gate.

   (7+8 marks)

2. Show that the singlet state
   $(|01\rangle - |10\rangle/)\sqrt{2}$ has density matrix representation ρ such that $tr(\rho^2) = 1$.

   (10 marks)

3. What is the classical lower bound on the number of function evaluations required in the Deustch-Josza problem? How is this achieved using a single quantum evaluation of the function in a quantum circuit? Explain the operation and analysis.

   (3+7 marks)

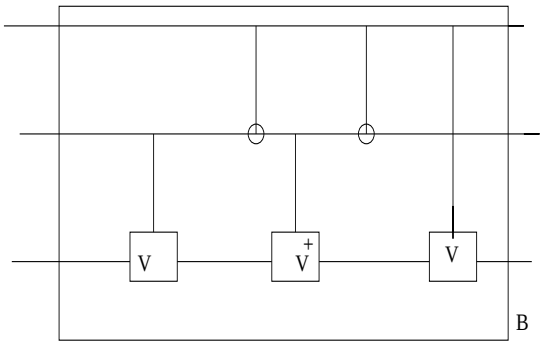
4. Show that two distinct one-qubit states may have the same density matrix representation.

   (10 marks)

5. Given that A, B, C are single qubit unitary operators (corresponding to 2 X 2 unitary matrices) and that $ABC = I$, show how you would realize the quantum circuit on 2 qubits performing the $C-U$ (controlled-U) operation where $U = AXBXC$ and X is the Pauli operator corresponding to the $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ matrix.

   (10 marks)

6. Consider the circuit B below where V is a unitary operator. Show that viewing B as a black box circuit with 3 qubits input-output, B realizes C^2-X when $V = (1 - \imath)(I + \imath X)/2$.



What is the significance of this result?

(10 marks)

7. Show that the 2-qubit state $(|00\rangle + |11\rangle)/\sqrt{2}$ can not be written as a tensor product of 2 pure single qubit states.

(10 marks)

8. Design a quantum circuit for teleportation in which all measurements are done with respect to the Bell Basis.

(10 marks)

9. Assuming that any 2×2 unitary matrix U has a representation in the form
$e^{\imath\alpha} \begin{pmatrix} a & b \\ -\bar{b} & \bar{a} \end{pmatrix}$ where $\alpha \in$ R and $a, b \in$ C, show that U can also be written as $e^{\imath\alpha}[I\cos(\theta/2) - \imath\sin(\theta/2)\hat{\sigma}.\hat{n}]$, where $0 \leq \theta \leq \pi, \hat{n}$is a unit vector and $\hat{\sigma} = \hat{\imath}X + \hat{\jmath}Y + \hat{k}Z$ for pauli matrices X,Y and Z.
Find α, θ, and $\hat{n}$ for H and S matrices.

(7+8 marks)

10. Given a GHZ state, show how an EPR pair state can be constructed.

(10 marks)

11. As soon as Alice and Bob receive one qubit each of a singlet state $(|01\rangle - |10\rangle)/\sqrt{2}$ they perform measurements Z and $(Z - X)/\sqrt{2}$ respectively, getting values Q and T respectively. What is the value of $\langle QT\rangle$?

(10 marks)