Quantum Computing and Quantum Information Processing, CS60014, Spring 2007

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Assignment #6, submission by 14/04/07

1. State the four postulates of quantum mechanics for quantum computation in terms of density operators where POVM measurements are used. What is the significance of POVM vis-a-vis projective measurements.

2. Show how POVM measurements are used in the proof of Holevo's bound using the three subsystems: the preparation system P, the quantum system Q and the measuring system M.

3. Show that the von Neumann entropy $S(\rho^{AB})$ of the tensor product of two separable states designated by $\rho^{AB} = \rho^A \otimes \rho^B$ is $S(\rho^A) + S(\rho^B)$.

4. Give an example demonstrating that conditional entropies can be negative.

5. Establish the subadditivity property of von Neumann entropy using Klein's inequality. When does equality hold? Give an example where the inequality holds strictly.

6. Derive the triangle inequality for von Neumann entropy. Generate an example that satisfies the inequality strictly.

7. Show that for a composite system ABC, $S(A|B,C) \leq S(A|B)$. Generate an example satisfying this inequality strictly.

8. Show that $S(A:B) \leq S(A:B,C)$ for composite system ABC. Characterize conditions under which the euality holds.

9. If ϵ is a trace-preserving quantum operation on system B of a composite system AB, then show that $S(A':B') \leq S(A,B)$.

10. Illustrate Holevo's theorem with non-trivial examples.