# Sixth Week

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We know that $P$ is not the same class as $DSPACE(n)$. To prove this result we use padding as well as the space hierarchy theorem. The assumption $P \subseteq DSPACE(n)$ implies $P = PSPACE$ (prove using a padding argument). So, assuming that $P$ and $DSPACE(n)$ are identical (for the sake of contradiction), implies $P = PSPACE = DSPACE(n)$.

**Exercise 1** *Show that $NP \neq NSPACE(n)$.*

**Exercise 2** *Show that $P = NP$ implies $DEXT = NEXT$.*

**Exercise 3**

**Exercise 4**

For the $L_{cfe}$ problem note that rule set 4 fixes the basis t=0, requiring all the $p(n)$ cells to have symbols of the initial ID for input $x$. Rule set 2 is the generic deterministic execution, giving $W = f(X, Y, Z)$ from three consecutive cells having $X, Y, Z$ at time $t - 1$ to the cell at $t$ getting $W$. Rules set 3 is used we deal with the last or the first cell ! The induction on $t$ requires showing for all $i$ in [1,$p(n)$] that $A_{W,i,t}$ derives the empty string if and only if $W$ is the $i$th symbol of the ID at time $t$. For the 'only if' part, the empty string must be derived by each of the three non-terminals in rule 2. Further, by induction hypothesis, $X, Y, Z$ must be in the three positions at time $t - 1$, and by the machine's execution, $W$ must in the $i$th place at time $t$, as given by $f(X, Y, Z)$. For the 'if part', $W$ is at the $i$th position at time $t$ and therefore, $X, Y, Z$ are in appropriate positions at time $t - 1$ given by $W = f(X, Y, Z)$. [Note that it does not matter even if $f$ is an onto funcction.] But by induction, the three non-terminals on the RHS of rule 2 derive empty strings, thereby forcing the LHS to also do so. Rest follows from rule one, proving P-completeness of $L_{cfe}$.