Advances in Algorithms: CS60011, Autumn 2003

Examples of perfect and non-perfect graphs;
Computing maximum independent sets in trees and polygons.

We give a few examples of graphs, some perfect and some non-perfect. We consider *trees*; trees are perfect graphs. Why? We consider computation of large independent sets for perfect graphs and also for a geometric problem.

1 Examples of graphs

We consider the properties of graphs defined earlier and study some examples. We wish to find $\alpha(G)$, $k(G)$, $\omega(G)z$ and $\chi(G)$ for the graphs in the figures. The values for the kite are Figure 1.1 are $\alpha = 3$, $k = 4$, $\omega = 2$ and $\chi = 3$. The values for the second graph (chair) are $\alpha = 3$, $k = 3$, $\omega = 3$ and χ=3. The first graph is a 6-vertex graph violating the perfect graph conditions, whereas the second graph is a 7-vertex graph in which both the equalities hold. Is this a perfect graph? Why?

Show that trees are perfect graphs and so are interval graphs.

2 Computing large independent sets

Now we consider the computation of large independent sets in special cases. First we consider trees and then we consider hidden sets in simple polygons.

2.1 Computing large independent sets in trees

. The first choice in Figure 2.2 gives a bigger independent set (shown ticked with red) whereas the second choice gives a smaller set (shown ticked with red).

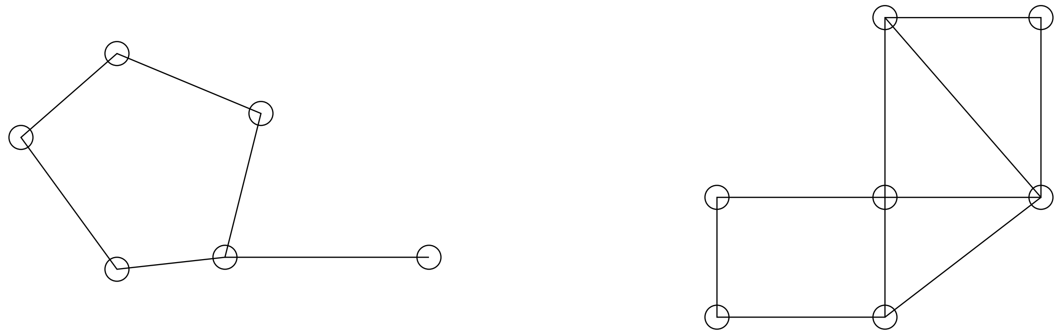

Figure 1.1: A kite and a chair

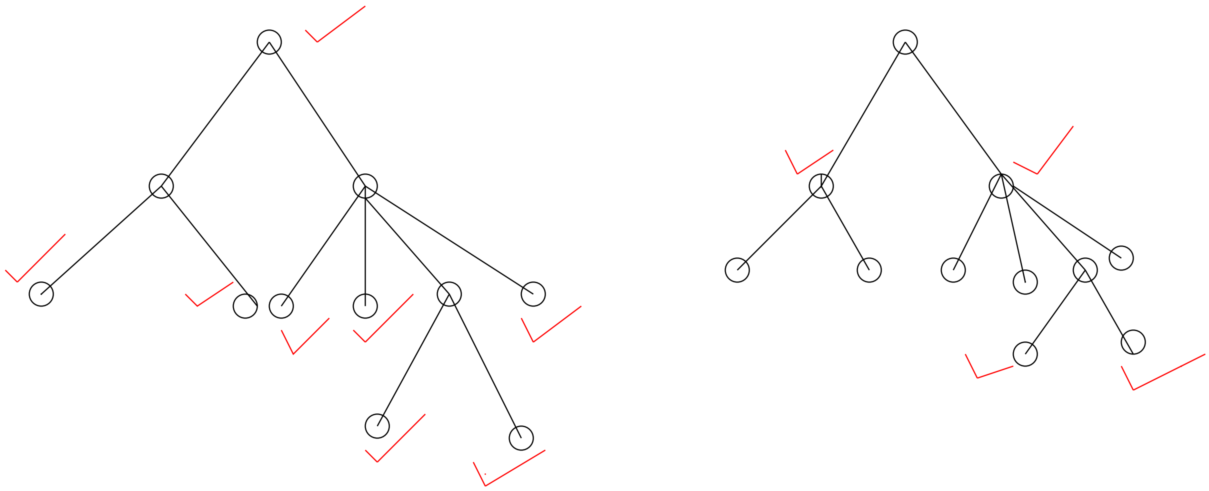

Figure 2.2: The root may or may not be in the independent set.

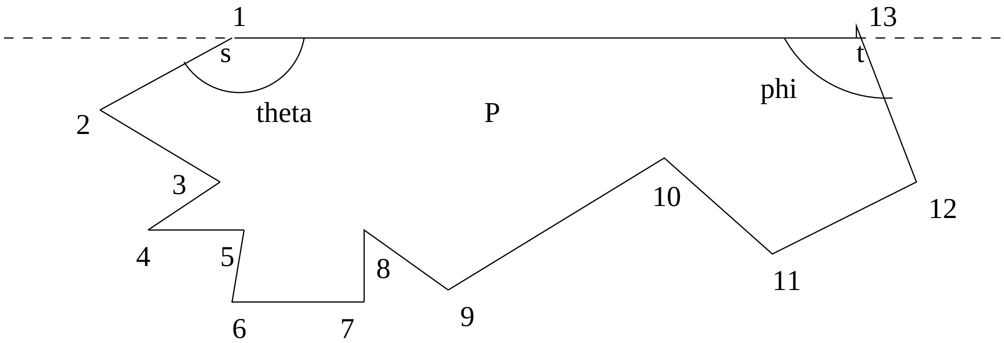

Figure 2.3: Weak visibility polygon: weakly visible from a convex edge.

2.2 Computing hidden sets in simple polygons

Finally, we consider polygons. Let P be a polygon such that (i) θ and ϕ are angles less than 180 degrees each (See Figure 2.3), (ii) for all $p \in bd(P)$ there exists $q \in \overline{st}$ such that $\overline{qp} \bigcap ext(P) = \oint$. Here $bd(P)$ is the boundary and $ext(P)$ is the exterior of the simple polygon P. Such simple polygons are called *weak visibility polygons* or polygons *weakly visible* from a *convex* edge *st*. The edge *st* is a convex edge in simple polygon P because the internal angles at s and t are less than 180 degrees. A hidden set H is a subset of the set of vertices of P such that for all $u, v \in H$, $\overline{uv} \bigcap ext(P) <> \oint$. Such a pair of vertices are not mutually *visible* to each other inside P. We wish to compute the maximum cardinality hidden set for a given n-vertex simple polygon P (weak visibility polygon) with properties (i) and (ii). Actually, this is a sort of a packing problem and we can design some heuristics.

Hidden sets are independent sets in the *visibility graph* of the polygon P. We define the *visibility graph* $G_P(V, E)$ of P as the graph with (i) vertex set V, the set of all vertices of P and (ii) undirected edge set E of pairs of mutually visible vertices in P.

Finding the largest hidden set for a general simple polygon in known to be NP-hard; the problem has a polynomial time algorithm for our class of weak visibility polygons. Examples of hidden sets for the polygon in Figure 2.3 are $\{2, 4, 6, 9, 11\}$ and $\{12, 8, 4\}$. What is the cardinality of the largest hidden set in this polygon? Why?

Are visibility graphs of weak visibility polygons (as defined above) perfect graphs? If so, are they chordal? These visibility graphs surely have Hamiltonian cycles.