Advances in Algorithms: CS60001, Autumn 2004
End-Semester Examination

November 25, 9 am to 12 noon, 2004, Thursday
Answer as many as possible.
Marks 100

1. It is known that the problem of deciding whether an undirected graph has a clique of size one-thirds the cardinality of its vertex set, is NP-complete. Show that the following problem is NP-complete: given undirected graphs G and H, determine whether H is isomorphic to an induced subgraph of G. [An induced subgraph G' of a graph G has only those edges of G that connect vertices of G'. Two graphs G_1 and G_2 are isomorphic if they have the same number n of vertices and if there is a permutation p of vertices in G_1 such that $\{u, v\}$ is an edge in G_1 if and only if $\{p(u), p(v)\}$ is an edge in G_2.] (5 marks)

2. Consider a set S of n vertices and seven distinct subsets of S, of five vertices each. Show that there exists a colouring of the n vertices in S using two colours RED and BLUE such that each of the seven sets has a vertex with RED colour as well as a vertex with BLUE colour. (10 marks)

3. Show that two logspace reductions can be cascaded into a single logspace reduction. (10 marks)

4. Show that CIRSAT is logspace reducible to SAT and NAESAT. (10 marks)

5. Show that 2SAT is in NP as well as in P. Is it also in NL? Why? Explain. (5+5+5 marks)

6. Suppose you use a Monte Carlo algorithm for computing the minimum cut in an undirected graph with some non-zero probability. How can you verify efficiently whether the run indeed gave a minimum cut? Explain clearly. (10 marks)

7. Design an $O(n \log n)$ time, divide-and-conquer algorithm for computing the convex hull of n points in the plane. (10 marks)

8. Consider the n vertex hypergraph colouring problem with red and blue colours where a proper colouring of the vertices is one that colours not all vertices red

or not all vertices blue in each hyperedge. Suppose $i \leq n$ vertices are already coloured. Consider *red* (*blue*) hyperedges, all of whose coloured vertices are coloured red (blue). Suppose the number of yet uncoloured vertices in the r red hyperedges are $n_1, n_2, ...n_r$, the number of yet uncoloured vertices in the b blue hyperedges are $m_1, m_2, ...m_b$, and the number of vertices in the u other hyperedges (with no vertices coloured so far) are $k_1, k_2, ...k_u$. Estimate an upper bound on the failure probability of a random colouring algorithm. (10 marks).

9. Consider the incremental randomized computation of the planar map of n line segments where the insertion sequence is a random permutation of the segments. If $j \leq n$ segments have already been inserted then show that the expected number of intersections so far in the current planar map is $O(\frac{kj^2}{n^2})$ where k is the total number of intersections between the n segments. (10 marks)

10. Given an unsorted sequence $a(1), a(2), .., a(n)$ of n integers, we wish to determine the length $c(i), 1 \leq i \leq n$, of the longest increasing subsequence within the sequence $a(1), a(2), ..., a(i)$ ending at $a(i)$. Design an $O(n \log n)$ algorithm for computing $c(i), 1 \leq i \leq n$. (10 marks)

11. Design a randomized version of a quicksort kind of algorithm that runs in $O(n \log n)$ time with probability away from 1 by at most an inverse polynomial in the size of the input. (10 marks)

12. Show that the diameter of a set of n points in the plane can be computed in $O(n \log n)$ time. (10 marks)

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