

# Indian Institute of Technology, Kharagpur

Date of Exam.: .02.15 (FN/AN) Time: 2 Hrs. Full Marks: 30 No. of Students: 300  
Mid (Spring) Semester Examination 2015 Department: Mathematics  
Subject No. MA20106 Subject Name : Probability & Stochastic Processes  
II yr B.Tech. EE/E&ECE/IE/MT

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## Instructions:

(i) Use of calculator and Statistical tables is allowed. All the notations are standard and no query or doubts will be entertained. If any data/statement is missing, identify it in your answer script.

(ii) Answer **All** questions.

(iii) All parts of a question **Must Be** answered at **One Place**.

1. (i) A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coin at random, and when he flips it, it shows heads. What is the probability that it is the fair coin?  
(ii) Suppose that he flips the same coin a second time and again it shows heads. Now what is the probability that it is the fair coin? (iii) Suppose that he flips the same coin a third time and it show tails. Now what is the probability that it is the fair coin ? [3]

2. A spacecraft has 30000 components. The probability of any one component being defective is  $10^{-5}$ . The mission will be in danger if 6 or more component become defective. Find the probability of such an event. [2]

3. Your teachers Dr. Nitin Gupta and Dr. Sourav Mukhopadhyay decide to play the following game. They toss a fair coin. If head appears, then Dr. Gupta gives one rupee to Dr. Mukhopadhyay and if tail appears, Dr. Mukhopadhyay gives one ruppee to Dr. Gupta. They continue this game till the 1000th toss and they both end up with the equal amount og money in their pockets. Assuming that the outcomes of all the tosses are independent, and Dr. Mukhopadhyay had Rs. 1000 and Dr. Gupta had Rs. 1600 in their pockets before the game started, calculate the probability that Dr. Mukhopadhyay never had more money in his pocket than Dr. Gupta during the entire game. [4]

4. Let  $X$  is a discrete type random variable with PMF as  $P(X = x) = C_x^{r+x-1} p^r (1-p)^x$ ,  $x = 0, 1, 2, \dots$ , where  $0 < p < 1$  and  $r > 0$  is an integer.

(a) Find the MGF of  $X$ .

(b) Let  $Y = 2pX$ . Show that the MGF of  $Y$  converges to a Chi-square random variable with  $2r$  degrees of freedom, that is, Show that

$$\lim_{p \rightarrow 0} M_Y(t) = (1 - 2t)^{-r}, |t| < \frac{1}{2}. \quad [4]$$

5. (a) Suppose the probability law of a random variable  $X$ , which is neither discrete nor continuous is given by

$$f(x) = \begin{cases} 0.2 & \text{if } x = 0 \\ 0.3 & \text{if } x = 1 \\ Ke^{-x} & \text{if } 1 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

(i) Find the constant  $K$ , (ii) determine  $E(X)$ , (iii) determine the m.g.f of  $X$ .

- (b) Suppose  $X_1, X_2, \dots, X_{10}$  are independent geometric random variable with same parameter  $p = 0.8$ . Let  $Y = X_1 + X_2 + \dots + X_{10}$ . Write the p.m.f of  $Y$ . [3+2]

6. (a) A fair die is tossed. Let  $X$  be a random variable which takes the value 1 if the result is an odd number, 2 if the result is an even number. Find the probability mass function (p.m.f) and the moment generating function (m.g.f) of  $X$ . [2+3]

(b) Let  $X$  be a continuous random variable with pdf

$$f(x) = \begin{cases} px, & 0 < x \leq 1, \\ p, & 1 < x \leq 2, \\ p(3-x), & 2 < x < 3, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Find the value of  $p$ . (ii) Find the cdf of  $X$ . (iii) Suppose  $X_1, X_2, X_3$  are three independent observations of  $X$ . What is the probability that exactly one of them is greater than  $3/2$ ?

8. (a) A battery cell is labelled as good if it works for at least 300 days in a clock, otherwise it is labelled as bad. Three manufacturers,  $A, B$  and  $C$  make cells with probability of making good cell as 0.95, 0.90 and 0.80 respectively. Three identical clocks are selected and cells made by  $A, B$  and  $C$  are used in clock number 1, 2 and 3 respectively. Let  $X$  be the total number of clocks working after 300 days. Find the probability mass function (p.m.f.) of  $X$ .

(b) Let  $X$  be a continuous random variable with the pdf

$$f(x) = \begin{cases} \frac{bx}{(1+x^2)}, & \sqrt{e-1} < x < \sqrt{e^2-1}, \\ 0, & \text{otherwise.} \end{cases}$$

Find  $b, P(2 < X^2 < 2.5)$  and  $V(X)$ .

- (c) Suppose that when a transistor of a certain type is subjected to accelerated life test, the life time  $X$  (in weeks) has a gamma distribution with mean 24 weeks and standard deviation 12 weeks. What is the probability that a transistor will last at most 24 weeks?

[3+2+2]

————The End————