1) ⓐ We can show all the outcomes in the form of a points table as follows:

S No.	Club F_1	Points of F_1	Points of F_2	league champ
1	3W, 0L	9	0	F_1
2	2W, 1D	7	1	F_1
3	2W, 1L	6	3	F_1
4	1W, 2D	5	2	F_1
5	1W, 1D, 1L	4	4	Tie
6	1W, 2L	3	6	F_2
7	3D	3	3	Tie
8	2D, 1L	2	5	F_2
9	1D, 2L	1	7	F_2
10	3L	0	9	F_2

W → win for F_1, D → draw for F_1, L → loss for F_1

(i) $P(F_1 \text{ wins the league}) = P(\{1\}) + P(\{2\}) + P(\{3\}) + P(\{4\})$

$$= \left(\tfrac{1}{2}\right)^3 + \binom{3}{2}\left(\tfrac{1}{2}\right)^2 \cdot \tfrac{1}{8} + \binom{3}{2}\left(\tfrac{1}{2}\right)^2 \cdot \tfrac{3}{8} + \binom{3}{1}\left(\tfrac{1}{2}\right)\left(\tfrac{1}{8}\right)^2$$

$$= \frac{67}{128}$$ 1M

(ii) $P(F_2 \text{ wins the league}) = P(\{6\}) + P(\{8\}) + P(\{9\}) + P(\{10\})$

$$= \binom{3}{2}\left(\tfrac{1}{2}\right)\left(\tfrac{3}{8}\right)^2 + \binom{3}{2}\left(\tfrac{1}{8}\right)^2\left(\tfrac{3}{8}\right) + \binom{3}{2}\left(\tfrac{1}{8}\right)\left(\tfrac{3}{8}\right)^2 + \left(\tfrac{3}{8}\right)^3$$

$$= \frac{171}{512}$$ 1M

(iii) $P(\text{Tie}) = 1 - \frac{67}{128} - \frac{171}{512} = \frac{73}{512}$ 1M

1.) (b)

$B_0 \rightarrow$ 0 defectives in the vial

$B_1 \longrightarrow$ 1 defective in the vial

$B_2 \longrightarrow$ 2 defectives in the vial

$B_3 \longrightarrow$ 3 defectives in the vial

$A \rightarrow$ two IC's are good

$P(B_0) = \frac{1}{3}$, $P(B_1) = \frac{1}{4}$, $P(B_2) = \frac{1}{4}$, $P(B_3) = \frac{1}{6}$

$P(A|B_0) = 1$, $P(A|B_1) = \dfrac{{}^4C_2}{{}^5C_2} = \dfrac{3}{5}$

$P(A|B_2) = \dfrac{{}^3C_2}{{}^5C_2} = \dfrac{3}{10}$

$P(A|B_3) = \dfrac{{}^2C_2}{{}^5C_2} = \dfrac{1}{10}$

$$P(B_0|A) = \frac{P(A|B_0)\,P(B_0)}{\sum_{i=0}^{3} P(A|B_i)\cdot P(B_i)} = \frac{40}{69}$$

$= 0.5797$ 2M

(2)

$X \sim Geo(0.8)$

$P(X > 25 \mid X > 20)$

$= P(X > 20+5 \mid X > 20)$

$= P(X \geq 5)$

$= 1 - P(X \leq 4)$

$= 1 - 0.9984$

$= 0.0016$ (1M)

3)

$$M_Y(t) = p^r (1 - qe^t)^{-r}$$

$$M_Y(t) = \left(\frac{2e^t}{3 - e^t} \right)^4$$

$$= \left(\frac{\frac{2}{3} e^t}{1 - \frac{e^t}{3}} \right)^4$$

$$Y = X + r$$

$$X \sim NB\left(4, \tfrac{2}{3}\right)$$

$$P(Y = K) = {}^{K-1}C_3 \left(\tfrac{1}{3}\right)^{K-4} \left(\tfrac{2}{3}\right)^4$$

$$P(5 \le X \le 7) = \sum_{K=5}^{7} {}^{K-1}C_3 \left(\tfrac{1}{3}\right)^{K-4} \left(\tfrac{2}{3}\right)^4$$

$$= \frac{1376}{2187}$$

$$= 0.629$$

$$E(X) = \frac{r}{p} = 6$$

2M

④

$$P(X=1) = F(1) - F(1-)$$

$$= 1 - 1$$

$$= 0$$

$$P\left(-2 < X \le \tfrac{1}{2}\right)$$

$$= F\left(\tfrac{1}{2}\right) - F(-2)$$

$$= \frac{3}{4} - 0$$

$$= \frac{3}{4}$$

Since for mixed type r.v.

$P(X=0) = \frac{1}{2}$ and for interval $0 < x < 1$ density is $\frac{1}{2}$

$$E(X) = 0.X\,\frac{1}{2} + \int_0^1 x \cdot \frac{1}{2}\,dx$$

$$= \frac{1}{4}$$

2M