Solution of Class Test (Probability and Stochastic processes held on 16/04/15) Set-1

1. A number X_0 is first chosen at random from the integers $1, 2, 3, 4, 5$. Let X_n be a number chosen at random from the integers $1, 2, ..., X_{n-1}$ for all $n = 1, 2, 3, ...$ Find the transition matrix for $\{X_n\}_{n=0,1,2,...}$. Identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
Ans: Order 5,

P =

	1	2	3	4	5	6	7
1	1	0	0	0	0		
2	1/2	1/2	0	0	0		
3	1/3	1/3	1/3	0	0		
4	1/4	1/4	1/4	1/4	0		
5	1/5	1/5	1/5	1/5	1/5		
6							
7							

2. A company has three machines. Each day, independent of each other, a machine breaks down with probability 1/2. Each night, there is one repairperson who can repair at most one machine. Let X_n be the number of machines available at the beginning of the n-th day. Find the transition matrix for the chain $\{X_n\}_{n=0,1,2,...}$. Identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
Ans: Order 3,

	1	2	3	4	5
1	1/2	1/2	0		
2	1/4	1/2	1/4		
3	1/8	3/8	1/2		
4					
5					

3. For which of the following transition probability matrices is the corresponding Markov chain irreducible?

$$(a)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (b)\begin{pmatrix} 0.2 & 0.0 & 0.8 \\ 0.6 & 0.2 & 0.2 \\ 0.8 & 0.0 & 0.2 \end{pmatrix}, \quad (c)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(i) Only (a), (ii) Only (b), (iii) Only (c), (iv) both (a) and (b),
(v) both (a) and (c), (vi) both (b) and (c), (vii) all.

Ans [(i)]

4. Let $\{X_n\}_{n=0,1,2,\ldots}$ be the MC with state space $S = \{1, 2, 3\}$ and the TPM

$$P = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}.$$

Find the limiting distribution of the Markov chain if it exists. State the conditions for existence.

$\pi_1 = [\quad 2/5 \quad]$, $\pi_2 = [\quad 1/5 \quad]$, $\pi_3 = [\quad 2/5 \quad]$.

5. In each game, a gambler wins the money he bets with probability 2/3 and loses with probability 1/3. If he has less than Rs. 3, he will bet all he has. Otherwise, since his goal is to have Rs. 5, he will only bet the difference between Rs. 5 and what he has. He continues to bet until he either has Rs. 0 or Rs. 5. Let X_n be the amount he has immediately after the n-th bet. Construct the TPM of the MC, identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
Ans: Order 6,

P =

	0	1	2	3	4	5	6
0	1	0	0	0	0	0	
1	1/3	0	2/3	0	0	0	
2	1/3	0	0	0	2/3	0	
3	0	1/3	0	0	0	2/3	
4	0	0	0	1/3	0	2/3	
5	0	0	0	0	0	1	
6							

For the above MC identify the states as indicated below:

- Identify absorbing states {0 & 5}
- Identify transient states {1, 2, 3, 4}
- Identify recurrent states {0 & 5}

Solution of Class Test (Probability and Stochastic processes held on 16/04/15) Set-2

1. A company has three machines. Each day, independent of each other, a machine breaks down with probability 1/3. Each night, there is one repairperson who can repair at most one machine. Let X_n be the number of machines available at the beginning of the n-th day. Find the transition matrix for the chain $\{X_n\}_{n=0,1,2,\ldots}$. Identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
Ans: Order 3,

	1	2	3	4	5
1	1/3	2/3	0		
2	1/9	4/9	4/9		
3	1/27	2/9	20/27		
4					
5					

2. For which of the following transition probability matrices is the corresponding Markov chain irreducible?

$$(a)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (b)\begin{pmatrix} 0.2 & 0.0 & 0.8 \\ 0.6 & 0.2 & 0.2 \\ 0.8 & 0.0 & 0.2 \end{pmatrix}, \quad (c)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(i) Only (a), (ii) Only (b), (iii) Only (c), (iv) both (a) and (b),
(v) both (a) and (c), (vi) both (b) and (c), (vii) all.

Ans [(i)]

3. Let $\{X_n\}_{n=0,1,2,\ldots}$ be the MC with state space $S = \{1, 2, 3\}$ and the TPM

$$P = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}.$$

Find the limiting distribution of the Markov chain if it exists. State the conditions for existence.

$\pi_1 = [$ 5/12 $]$, $\pi_2 = [$ 1/4 $]$, $\pi_3 = [$ 1/3 $]$.

4. In each game, a gambler wins the money he bets with probability 1/5 and loses with probability 4/5. If he has less than Rs. 3, he will bet all he has. Otherwise, since his goal is to have Rs. 5, he will only bet the difference between Rs. 5 and what he has. He continues to bet until he either has Rs. 0 or Rs. 5. Let X_n be the amount he has immediately after the n-th bet. Construct the TPM of the MC, identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
Ans: Order 6,

P =

	0	1	2	3	4	5	6
0	1	0	0	0	0	0	
1	4/5	0	1/5	0	0	0	
2	4/5	0	0	0	1/5	0	
3	0	4/5	0	0	0	1/5	
4	0	0	0	4/5	0	1/5	
5	0	0	0	0	0	1	
6							

For the above MC identify the states as indicated below:

- Identify absorbing states {0 & 5}
- Identify transient states {1, 2, 3, 4}
- Identify recurrent states {0 & 5}

5. A number X_0 is first chosen at random from the integers $1, 2, 3, 4, 5$. Let X_n be a number chosen at random from the integers $1, 2, ..., X_{n-1}$ for all $n = 1, 2, 3, ...$ Find the transition matrix for $\{X_n\}_{n=0,1,2,...}$. Identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
Ans: Order 5,

P =

	1	2	3	4	5	6	7
1	1	0	0	0	0		
2	1/2	1/2	0	0	0		
3	1/3	1/3	1/3	0	0		
4	1/4	1/4	1/4	1/4	0		
5	1/5	1/5	1/5	1/5	1/5		
6							
7							

Solution of Class Test (Probability and Stochastic processes held on 16/04/15) Set-3

1. For which of the following transition probability matrices is the corresponding Markov chain irreducible?

$$(a)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (b)\begin{pmatrix} 0.2 & 0.0 & 0.8 \\ 0.6 & 0.2 & 0.2 \\ 0.8 & 0.0 & 0.2 \end{pmatrix}, \quad (c)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(i) Only (a), (ii) Only (b), (iii) Only (c), (iv) both (a) and (b),
(v) both (a) and (c), (vi) both (b) and (c), (vii) all.

Ans [(i)]

2. Let $\{X_n\}_{n=0,1,2,...}$ be the MC with state space $S = \{1, 2, 3\}$ and the TPM

$$P = \begin{bmatrix} 5/8 & 1/4 & 1/8 \\ 1/9 & 4/9 & 4/9 \\ 1/5 & 3/5 & 1/5 \end{bmatrix}.$$

Find the limiting distribution of the Markov chain if it exists. State the conditions for existence.

$\pi_1 = [$ 16/57 $]$, $\pi_2 = [$ 33/76 $]$, $\pi_3 = [$ 65/228 $]$.

3. In each game, a gambler wins the money he bets with probability 3/5 and loses with probability 2/5. If he has less than Rs. 3, he will bet all he has. Otherwise, since his goal is to have Rs. 5, he will only bet the difference between Rs. 5 and what he has. He continues to bet until he either has Rs. 0 or Rs. 5. Let X_n be the amount he has immediately after the n-th bet. Construct the TPM of the MC, identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
Ans: Order 6,

P =

	0	1	2	3	4	5	6
0	1	0	0	0	0	0	
1	2/5	0	3/5	0	0	0	
2	2/5	0	0	0	3/5	0	
3	0	2/5	0	0	0	3/5	
4	0	0	0	2/5	0	3/5	
5	0	0	0	0	0	1	
6							

For the above MC identify the states as indicated below:

- Identify absorbing states {0 & 5}
- Identify transient states {1, 2, 3, 4}
- Identify recurrent states {0 & 5 }

4. A number X_0 is first chosen at random from the integers $1, 2, 3, 4, 5$. Let X_n be a number chosen at random from the integers $1, 2, ..., X_{n-1}$ for all $n = 1, 2, 3, ...$ Find the transition matrix for $\{X_n\}_{n=0,1,2,...}$. Identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
   Ans: Order 5,

P =

	1	2	3	4	5	6	7
1	1	0	0	0	0		
2	1/2	1/2	0	0	0		
3	1/3	1/3	1/3	0	0		
4	1/4	1/4	1/4	1/4	0		
5	1/5	1/5	1/5	1/5	1/5		
6							
7							

5. A company has three machines. Each day, independent of each other, a machine breaks down with probability 1/4. Each night, there is one repairperson who can repair at most one machine. Let X_n be the number of machines available at the beginning of the n-th day. Find the transition matrix for the chain $\{X_n\}_{n=0,1,2,...}$. Identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
   Ans: Order 3,

	1	2	3	4	5
1	1/4	3/4	0		
2	1/16	3/8	9/16		
3	1/64	9/64	27/32		
4					
5					

Solution of Class Test (Probability and Stochastic processes held on 16/04/15) Set-4

1. Let $\{X_n\}_{n=0,1,2,...}$ be the MC with state space $S = \{1, 2, 3\}$ and the TPM

$$P = \begin{bmatrix} 5/8 & 1/4 & 1/8 \\ 1/10 & 3/10 & 3/5 \\ 1/5 & 3/5 & 1/5 \end{bmatrix}.$$

Find the limiting distribution of the Markov chain if it exists. State the conditions for existence.

$\pi_1 = [\quad 16/57 \quad]$, $\pi_2 = [\quad 22/57 \quad]$, $\pi_3 = [\quad 1/3 \quad]$.

2. In each game, a gambler wins the money he bets with probability 1/6 and loses with probability 5/6. If he has less than Rs. 3, he will bet all he has. Otherwise, since his goal is to have Rs. 5, he will only bet the difference between Rs. 5 and what he has. He continues to bet until he either has Rs. 0 or Rs. 5. Let X_n be the amount he has immediately after the n-th bet. Construct the TPM of the MC, identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
Ans: Order 6,

P =

	0	1	2	3	4	5	6
0	1	0	0	0	0	0	
1	5/6	0	1/6	0	0	0	
2	5/6	0	0	0	1/6	0	
3	0	5/6	0	0	0	1/6	
4	0	0	0	5/6	0	1/6	
5	0	0	0	0	0	1	
6							

For the above MC identify the states as indicated below:

- Identify absorbing states {0 & 5}
- Identify transient states {1, 2, 3, 4}
- Identify recurrent states {0 & 5}

3. A number X_0 is first chosen at random from the integers $1, 2, 3, 4, 5$. Let X_n be a number chosen at random from the integers $1, 2, ..., X_{n-1}$ for all $n = 1, 2, 3, ...$ Find the transition matrix for $\{X_n\}_{n=0,1,2,...}$. Identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
Ans: Order 5,

P =

	1	2	3	4	5	6	7
1	1	0	0	0	0		
2	1/2	1/2	0	0	0		
3	1/3	1/3	1/3	0	0		
4	1/4	1/4	1/4	1/4	0		
5	1/5	1/5	1/5	1/5	1/5		
6							
7							

4. A company has three machines. Each day, independent of each other, a machine breaks down with probability 2/3. Each night, there is one repairperson who can repair at most one machine. Let X_n be the number of machines available at the beginning of the n-th day. Find the transition matrix for the chain $\{X_n\}_{n=0,1,2,\ldots}$. Identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
Ans: Order 3,

	1	2	3	4	5
1	2/3	1/3	0		
2	4/9	4/9	1/9		
3	8/27	4/9	7/27		
4					
5					

5. For which of the following transition probability matrices is the corresponding Markov chain irreducible?

$$(a)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (b)\begin{pmatrix} 0.2 & 0.0 & 0.8 \\ 0.6 & 0.2 & 0.2 \\ 0.8 & 0.0 & 0.2 \end{pmatrix}, \quad (c)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(i) Only (a), (ii) Only (b), (iii) Only (c), (iv) both (a) and (b),
(v) both (a) and (c), (vi) both (b) and (c), (vii) all.

Ans [(i)]

Solution of Class Test (Probability and Stochastic processes held on 16/04/15) Set-5

1. In each game, a gambler wins the money he bets with probability 1/3 and loses with probability 2/3. If he has less than Rs. 3, he will bet all he has. Otherwise, since his goal is to have Rs. 5, he will only bet the difference between Rs. 5 and what he has. He continues to bet until he either has Rs. 0 or Rs. 5. Let X_n be the amount he has immediately after the n-th bet. Construct the TPM of the MC, identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
   Ans: Order 6,

P =

	0	1	2	3	4	5	6
0	1	0	0	0	0	0	
1	2/3	0	1/3	0	0	0	
2	2/3	0	0	0	1/3	0	
3	0	2/3	0	0	0	1/3	
4	0	0	0	2/3	0	1/3	
5	0	0	0	0	0	1	
6							

   For the above MC identify the states as indicated below:

   - Identify absorbing states {0 & 5}
   - Identify transient states {1, 2, 3, 4}
   - Identify recurrent states {0 & 5 }

2. A number X_0 is first chosen at random from the integers $1, 2, 3, 4, 5$. Let X_n be a number chosen at random from the integers $1, 2, ..., X_{n-1}$ for all $n = 1, 2, 3, ...$ Find the transition matrix for $\{X_n\}_{n=0,1,2,...}$. Identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
   Ans: Order 5,

P =

	1	2	3	4	5	6	7
1	1	0	0	0	0		
2	1/2	1/2	0	0	0		
3	1/3	1/3	1/3	0	0		
4	1/4	1/4	1/4	1/4	0		
5	1/5	1/5	1/5	1/5	1/5		
6							
7							

3. A company has three machines. Each day, independent of each other, a machine breaks down with probability 3/5. Each night, there is one repairperson who can repair at most one machine. Let X_n be the number of machines available at the beginning of the n-th day. Find the transition matrix for the chain $\{X_n\}_{n=0,1,2,...}$. Identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
Ans: Order 3,

	1	2	3	4	5
1	3/5	2/5	0		
2	9/25	12/25	4/25		
3	27/125	54/125	44/125		
4					
5					

4. For which of the following transition probability matrices is the corresponding Markov chain irreducible?

$$(a)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (b)\begin{pmatrix} 0.2 & 0.0 & 0.8 \\ 0.6 & 0.2 & 0.2 \\ 0.8 & 0.0 & 0.2 \end{pmatrix}, \quad (c)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(i) Only (a), (ii) Only (b), (iii) Only (c), (iv) both (a) and (b),
(v) both (a) and (c), (vi) both (b) and (c), (vii) all.

Ans [(i)]

5. Let $\{X_n\}_{n=0,1,2,...}$ be the MC with state space $S = \{1, 2, 3\}$ and the TPM

$$P = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 5/8 & 1/4 & 1/8 \\ 1/10 & 3/10 & 3/5 \end{bmatrix}.$$

Find the limiting distribution of the Markov chain if it exists. State the conditions for existence.

$\pi_1 = [\ 2/5\ ]$, $\pi_2 = [\ 4/15\ ]$, $\pi_3 = [\ 1/3\ ]$.

Solution of Class Test (Probability and Stochastic processes held on 16/04/15) Set-6

1. A company has three machines. Each day, independent of each other, a machine breaks down with probability 1/5. Each night, there is one repairperson who can repair at most one machine. Let X_n be the number of machines available at the beginning of the n-th day. Find the transition matrix for the chain $\{X_n\}_{n=0,1,2,...}$. Identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
Ans: Order 3,

	1	2	3	4	5
1	1/5	4/5	0		
2	1/25	8/25	16/25		
3	1/125	12/125	112/125		
4					
5					

2. For which of the following transition probability matrices is the corresponding Markov chain irreducible?

$$(a)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (b)\begin{pmatrix} 0.2 & 0.0 & 0.8 \\ 0.6 & 0.2 & 0.2 \\ 0.8 & 0.0 & 0.2 \end{pmatrix}, \quad (c)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(i) Only (a), (ii) Only (b), (iii) Only (c), (iv) both (a) and (b),
(v) both (a) and (c), (vi) both (b) and (c), (vii) all.

Ans [(i)]

3. A number X_0 is first chosen at random from the integers $1, 2, 3, 4, 5$. Let X_n be a number chosen at random from the integers $1, 2, ..., X_{n-1}$ for all $n = 1, 2, 3, ...$ Find the transition matrix for $\{X_n\}_{n=0,1,2,...}$. Identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
Ans: Order 5,

P =

	1	2	3	4	5	6	7
1	1	0	0	0	0		
2	1/2	1/2	0	0	0		
3	1/3	1/3	1/3	0	0		
4	1/4	1/4	1/4	1/4	0		
5	1/5	1/5	1/5	1/5	1/5		
6							
7							

4. Let $\{X_n\}_{n=0,1,2,...}$ be the MC with state space $S = \{1, 2, 3\}$ and the TPM

$$P = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 5/9 & 2/9 & 2/9 \\ 1/10 & 3/10 & 3/5 \end{bmatrix}.$$

Find the limiting distribution of the Markov chain if it exists. State the conditions for existence.

$\pi_1 = [$ 22/73 $]$, $\pi_2 = [$ 21/73 $]$, $\pi_3 = [$ 30/73 $]$.

5. In each game, a gambler wins the money he bets with probability 1/4 and loses with probability 3/4. If he has less than Rs. 3, he will bet all he has. Otherwise, since his goal is to have Rs. 5, he will only bet the difference between Rs. 5 and what he has. He continues to bet until he either has Rs. 0 or Rs. 5. Let X_n be the amount he has immediately after the n-th bet. Construct the TPM of the MC, identify the order of TPM and by choosing suitable order from the following table, fill the entries of the matrix in the space provided below.
Ans: Order 6,

P =

	0	1	2	3	4	5	6
0	1	0	0	0	0	0	
1	3/4	0	1/4	0	0	0	
2	3/4	0	0	0	1/4	0	
3	0	3/4	0	0	0	1/4	
4	0	0	0	3/4	0	1/4	
5	0	0	0	0	0	1	
6							

For the above MC identify the states as indicated below:

- Identify absorbing states {0 & 5}
- Identify transient states {1, 2, 3, 4}
- Identify recurrent states {0 , 5}