End Sem Solution

1. a) Let $c = [X < Y]$

$$P(C) = P(X < Y) = \iint\limits_{[(u,v):\, u < v]} f(u,v)\, du\, dv$$

$$= \int_0^\infty \left\{ \int_0^y 2e^{-u} e^{-2v}\, du \right\} dv$$

$$= \int_0^\infty 2e^{-2v} \left[-e^{-u}\right]_0^v dv$$

$$= \int_0^\infty 2e^{-2v} \left[1 - e^{-v}\right] dv$$

$$= \int_0^\infty 2e^{-2v} \left[1 - e^{-v}\right] dv$$

$$= \int_0^\infty 2e^{-2v}\, dv - \int_0^\infty 2e^{-3v}\, dv$$

$$= \left[-e^{-2v}\right]_0^\infty - \left[-\frac{2}{3} e^{-3v}\right]_0^\infty = 1 - \frac{2}{3} = \boxed{\frac{1}{3}}$$

1. b)

$$f_{X,Y}(x,y) = \begin{cases} 2 & \text{if } 0<x<y<1 \\ 0 & \text{otherwise.} \end{cases}$$

$$f_Y(y) = \int_0^y f_{X,Y}(x,y)\,dx$$

$$= \int_0^y 2\,dx = 2y$$

$$f_{X|Y=y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)} = \frac{2}{2y} = \frac{1}{y}$$

$$f_{X|Y=y}(x|y) = \begin{cases} \frac{1}{y} & 0<x<y<1 \\ 0 & \text{Otherwise} \end{cases}$$

$$E(X|Y=y) = \int_0^y x\, f_{X|Y}\,dx$$

$$= \int_0^y \frac{x}{y}\,dx = \frac{1}{y}\left[\frac{x^2}{2}\right]_0^y$$

$$= \frac{y}{2} \quad \text{- - - Ans}$$

1. c)

$$P(X=x, Y=y) = \begin{cases} \dfrac{2}{n(n+1)} & \text{if } x = 1, 2, \dots n;\ y = 1, 2, \dots x \\ 0 & \text{otherwise} \end{cases}$$

$$P(X=x) = \sum_{y=1}^{x} \frac{2}{n(n+1)} = \frac{2x}{n(n+1)}$$

$$P(Y=y \mid X=x) = \frac{P(X=x, Y=y)}{P(X=x)}$$

$$= \frac{1}{x}$$

$$E(Y \mid X=x) = \sum_{y=1}^{x} \frac{y}{x} = \frac{1}{x} \cdot \frac{x(x+1)}{2} = \boxed{\frac{x+1}{2}}$$

1. d) $(X, Y) \sim N(5, 10, 1, 25, \rho)$

$Y \mid X=5 \sim N(\mu, \sigma^2)$ where $\mu = \mu_Y + \rho \cdot \frac{\sigma_Y}{\sigma_X}(x - \mu_x)$

$= 10$

$\sigma^2 = \sigma_Y^2 (1-\rho^2)$

$= 25(1-\rho^2)$

$P(4 < Y < 16 \mid X=5) = 0.954$

$\Rightarrow P\left(\frac{-6}{5\sqrt{1-\rho^2}} < \frac{Y-10}{5\sqrt{1-\rho^2}} < \frac{6}{5\sqrt{1-\rho^2}}\right) = 0.954$

$\Rightarrow P\left(|Z| < \frac{6}{5\sqrt{1-\rho^2}}\right) = 0.954$

$\therefore P\left(Z < \frac{6}{5\sqrt{1-\rho^2}}\right) = \frac{0.954}{2} + 0.5$

$= 0.977$

$\Rightarrow \frac{6}{5\sqrt{1-\rho^2}} = 2$

$\Rightarrow \rho = \boxed{0.8}$

1.e)

$$P\{X_{n+3}=0 \mid X_{n+2}=1, X_{n+1}=1, X_n=0\}$$

$$= P\{X_{n+3}=0 \mid X_{n+2}=1\} \quad \text{[Markov Property]}$$

$$= P[X_1=0 \mid X_0=1]$$

$$= \boxed{\frac{1}{3}}$$

1. f)

$$P\{X_{n+2}=0, X_{n+1}=1 \mid X_n=1\}$$

$$= P\{X_{n+2}=0 \mid X_{n+1}=1, X_n=1\}\, P\{X_{n+1}=1 \mid X_n=1\}$$

$$= P\{X_{n+2}=0 \mid X_{n+1}=1\}\, P\{X_{n+1}=1 \mid X_n=1\}$$

$$= \frac{1}{3}\cdot\frac{2}{3} = \boxed{\frac{2}{9}}$$

1. g)

$$P\{X_0=1, X_1=0, X_2=2\}$$

$$= P\{X_2=2 \mid X_1=0, X_0=1\}\cdot P\{X_1=0, X_0=1\}$$

$$= P\{X_2=2 \mid X_1=0\}\cdot P\{X_1=0 \mid X_0=1\}\cdot P\{X_0=1\}$$

$$= p_{02}\cdot p_{10}\cdot 1$$

$$= (0.1)(0.3)(1)$$

$$= \boxed{0.03}$$

1 h) $P(X_2 = 2) = P\{X_2 = 2 \mid X_0 = 0\} \cdot P\{X_0 = 0\}$

$+ P\{X_2 = 2 \mid X_0 = 1\} \cdot P\{X_0 = 1\}$

$+ P\{X_2 = 2 \mid X_0 = 2\} \cdot P\{X_0 = 2\}$

$= p_{02}^{(2)} \cdot 0 + p_{12}^{(2)} \cdot 1 + p_{22}^{(2)} \cdot 0$

$= p_{12}^{(2)} = \boxed{0.35}$

$$P = \begin{array}{c} 0 \\ 1 \\ 2 \end{array} \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.4 & 0.1 & 0.5 \end{bmatrix}$$

$$P^2 = \begin{array}{c} 0 \\ 1 \\ 2 \end{array} \begin{bmatrix} 0.49 & 0.28 & 0.23 \\ 0.43 & 0.22 & 0.35 \\ 0.47 & 0.20 & 0.33 \end{bmatrix}$$

Alternative way

$\pi^{(0)} = [0, 1, 0]$

then $\pi^{(2)} = \pi^{(0)} P^2$

$$= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.49 & 0.28 & 0.23 \\ 0.43 & 0.22 & 0.35 \\ 0.47 & 0.20 & 0.33 \end{bmatrix}$$

$$= \begin{bmatrix} 0.43 & 0.22 & 0.35 \end{bmatrix}$$

$P\{X_2 = 0\} = 0.43$

$P\{X_2 = 1\} = 0.22$

$\boxed{P\{X_2 = 2\} = 0.35}$

2a) $P(\text{5 die all show a particular number}) = \left(\frac{1}{6}\right)^5$

Since there are 6 possible numbers, the probability of wining the game

$$= 6 \times \left(\frac{1}{6}\right)^5 = \frac{1}{6^4} = \frac{1}{1296}$$

gain to the person by wining j games $= 20j - 50$, $j = 0,1,2,3,4,5$

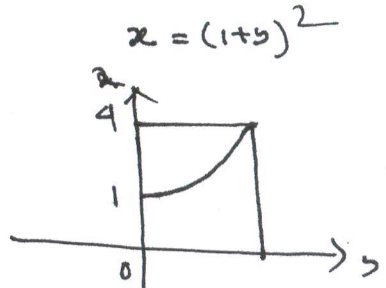
$$P(X = 20j - 50) = {}^5C_j \left(\frac{1}{1296}\right)^j \left(\frac{1295}{1296}\right)^{5-j} , \quad j = 0,1,2,\cdots 5$$

$$E(X) = 20 \sum_{j=0}^{5} j \, {}^5C_j \left(\frac{1}{1296}\right)^j \left(\frac{1295}{1296}\right)^{5-j} - 50$$

$$= 20 \times 5 \times \frac{1}{1296} - 50$$

$$= -49.92$$

2b) $X = (1+Y)^2$ $\quad \therefore y = \sqrt{x} - 1$



pdf of X is

$$f_X(x) = f_x(y) \left| \frac{dy}{dx} \right| = \begin{cases} \frac{\sqrt{x} - 1}{x} & 1 < x < 4 \\ 0 & \text{o.w.} \end{cases}$$

$$E(X) = \int_1^4 x \frac{\sqrt{x} - 1}{\sqrt{x}} \, dx = \frac{17}{6}$$

$$E(X^2) = \int_1^4 x^2 \frac{\sqrt{x} - 1}{\sqrt{x}} \, dx = 8.53$$

$$Var(X) = E(X^2) - (E(X))^2 = 0.5$$

2 c)

(i) $\int f_X(x)\,dx = 1 \Rightarrow c = \frac{2}{9}$

(ii) cdf of X is

$$F_X(x) = \begin{cases} 0 & x < 0 \\ x^2/9 & 0 \le x < 3 \\ 1 & x \geq 3 \end{cases}$$

(iii) $E(X) = \int_0^3 x \cdot \frac{2}{9}x\,dx = 2$

$E(X^2) = \int_0^3 x^2 \cdot \frac{2}{9}x\,dx = \frac{9}{2}$

$Var(X) = E(X^2) - (E(X))^2 = \frac{1}{2}$

(iv) $P(X \le m) = \frac{1}{2}$

$\Rightarrow \frac{m^2}{9} = 1/2 \Rightarrow m = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}}$

where m $\rightarrow$ median.

3)(a)

$n = 50$

$X_i \sim Poiss(\lambda)$ $\quad X_1, X_2, \cdots, X_n$ independent

$E(X_i) = \lambda = V(X_i)$ $\quad i = 1, 2, \cdots 50$

$\lambda = 0.03$

$$S_n = \sum_{i=1}^{50} X_i$$

$$E(S_n) = n\lambda = 50 \times 0.03 = 1.5$$

$$V(S_n) = V\left(\sum_{i=1}^{50} X_i\right) = 50 \times Var(X_i) = 50 \times 0.03 = 1.5$$

Using CLT

$$P(S_n \geq 3) = P\left[\frac{S_n - E(S_n)}{\sqrt{V(S_n)}} \geq \frac{3 - 1.5}{\sqrt{1.5}}\right]$$

$$= P(Z \geq 1.23), \text{when } Z = \frac{S_n - E(S_n)}{\sqrt{V(S_n)}} \sim N(0,1)$$

$$= 1 - \Phi(1.23)$$

$$= 0.1112$$

Exact Value

$$S_n = \sum_{i=1}^{50} X_i \sim P(1.5)$$

$$P(S_n \geq 3) = 1 - P(S_n < 3)$$

$$= 1 - [P(S_n = 0) + P(S_n = 1) + P(S_n = 2)]$$

$$= 1 - \left[\frac{e^{-1.5}(1.5)^0}{0!} + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!}\right]$$

$$= 0.1912$$

Using CLT $\quad P(S_n \geq 3) = 0.1112$

Exact value $\quad P(S_n \geq 3) = 0.1912$

(3b) $X \sim \text{Poisson}(\lambda)$, where $\lambda = 1$

$$E\left\{\frac{1}{1+X}\right\}$$

$$= \sum_{i=0}^{\infty} \frac{1}{(i+1)}\left(e^{-\lambda}\frac{\lambda^i}{\lfloor i}\right)$$

$$= e^{-\lambda}\sum_{i=0}^{\infty} \frac{\lambda^i}{\lfloor i+1}$$

$$= \frac{e^{-\lambda}}{\lambda}\sum_{i=0}^{\infty} \frac{\lambda^{i+1}}{\lfloor i+1}$$

$$= \frac{e^{-\lambda}}{\lambda}\sum_{i=1}^{\infty} \frac{\lambda^i}{\lfloor i}$$

$$= \frac{e^{-\lambda}}{\lambda}\left[\sum_{i=0}^{\infty} \frac{\lambda^i}{\lfloor i} - \frac{\lambda^0}{\lfloor 0}\right]$$

$$= \frac{e^{-\lambda}}{\lambda}\left[e^{\lambda} - 1\right]$$

$$= \frac{e^{-\lambda+\lambda}}{\lambda} - \frac{e^{-\lambda}}{\lambda}$$

$$= \frac{1}{\lambda} - \frac{e^{-\lambda}}{\lambda}$$

$$= 1 - e^{-1} \qquad [\because \lambda = 1]$$

$$= 1 - \frac{1}{e}$$

$$= 0.6321$$

(3c) $n = 100$, $p = 0.1$

$P(12 < X < 14)$ with normal distribution approximation

$\mu = np = 10$

$\sigma^2 = npq = 100 \times 0.1 \times 0.9$
$= 9$

$\Rightarrow \sigma = 3$

$$P(12 < X < 14) = P\left(\frac{12-10}{3} < Z < \frac{14-10}{3}\right)$$
$$= P\left(\frac{2}{3} < Z < \frac{4}{3}\right)$$
$$= P(0.6667 < Z < 1.333)$$
$$= \Phi(1.33) - \Phi(0.6667)$$
$$= 0.90824 - 0.74857$$
$$= 0.15967$$

With Continuity Correction

$$P(12 < X < 14) = P(X = 13)$$
$$\simeq P(12.5 \leq X \leq 13.5)$$
$$= P\left(\frac{12.5-10}{3} \leq Z \leq \frac{13.5-10}{3}\right)$$
$$= P(0.08333 \leq Z \leq 1.1666)$$
$$= \Phi(1.16) - \Phi(0.83)$$
$$= 0.8770 - 0.7967$$
$$= 0.0803$$

Exact value

$$P(12 < X < 14) = P(X = 13)$$
$$= \binom{100}{13} (0.1)^{13} (0.09)^{87}$$
$$= 0.0743020$$

4. a) $X \sim NB(r, p)$

$$p(X = x) = {}^{-r}C_i \, p^r (-q)^i \qquad i = 0, 1, 2, \dots, \infty$$

Moment generatting function of X

$$= M_X(t) = E(e^{tx})$$

$$= \sum_{i=0}^{\infty} e^{ti} \, {}^{-r}C_i \, p^r (-q)^i$$

$$= p^r \sum_{i=0}^{\infty} {}^{-r}C_i \, (-q e^t)^i$$

$$= p^r (1 - qe^t)^{-r}$$

$$= \left(\frac{p}{1 - qe^t}\right)^r$$

$$E(X) = M_X'(0) = p^r (-r) \frac{1}{(1 - qe^t)^{r+1}} (-q e^t) \Big|_{t=0}$$

$$= \frac{p^r r \cdot q}{(1-q)^{r+1}} = \frac{rq}{p}$$

$$E(X^2) = M_X''(0) = \frac{(1-qe^t)^{r+1} p^r r q e^t - p^r r q e^t (r+1)(1-qe^t)^r (-qe^t)}{(1-qe^t)^{2(r+1)}} \Bigg|_{t=0}$$

$$= \frac{(1-q)^{r+1} p^r r q + p^r r q^2 (r+1)(1-q)^r}{(1-q)^{2(r+1)}}$$

$$= \frac{rq}{p} + \frac{r^2 q^2}{p^2} + \frac{rq^2}{p^2}$$

$$Var(X) = E(X^2) - E^2(X)$$

$$= \frac{rq}{p} + \frac{r^2q^2}{p^2} + \frac{rq^2}{p^2} - \frac{r^2q^2}{p^2}$$

$$= \frac{rq}{p}\left(1 + \frac{q}{p}\right) = \frac{rq}{p^2}$$

4. b) The memoryless property of geometric distribution is states as

$P(X > s+t \mid X > s) = P(X \geq t)$ where $X \sim Geo(p)$

$\Rightarrow$ $X \sim Geo(p)$

$P(X=i) = q^i p \quad i = 0, 1, 2, \ldots \infty$

$P(X > K) = 1 - P(X \leq K)$

$= 1 - P(q^0 + q^1 + \ldots + q^K)$

$= 1 - p \dfrac{1-q^{K+1}}{1-q} = q^{K+1}$

$P(X > s+t \mid X > s)$

$= \dfrac{P(X > s+t)}{P(X > s)}$

$= \dfrac{q^{s+t+1}}{q^{s+1}}$

$= q^t = P(X \geq t)$

4c) $X \sim N(0,1)$

$Y = X^2 = g(X)$

where $g(x) = x^2$ $\quad g'(x) = 2x > 0$ if $x > 0$, < 0 if $x < 0$

This function is continuous but strictly increasing on $(0, \infty)$ and strictly decreasing on $(-\infty, 0)$. Further the function is not one to one since $g(x) = g(-x)$

Therefore we cannot apply following result directly :-

$$\boxed{f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right|} \quad X$$

method :-

It is obvious that

$$F_Y(y) = P(Y \le y) = P(X^2 \le y) = 0 \text{ for } y < 0$$

Suppose $y > 0$. Then

$$F_Y(y) = P(Y \le y) = P(X^2 \le y)$$

$$= P(|X| \le \sqrt{y})$$

$$= P(-\sqrt{y} \le X \le \sqrt{y})$$

$$= \int_{-\sqrt{y}}^{\sqrt{y}} \phi(x)\, dx$$

Where $\phi(x)$ denotes the standard normal probability density function. From the symmetry of the probability density function $\phi(x)$, we get that

$$F_Y(y) = 2 \int_0^{\sqrt{y}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$$

$$= \int_0^{y} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z} z^{-1/2} dz$$

It is easy to see that $f_Y(0) = 0$. The above derivation shows that the random variable Y has a probability density function and it is given by

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y} y^{-1/2} \quad \text{for } y > 0$$

$$= 0 \quad \text{for } y \leq 0$$

Recalling that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$, we can rewrite the function $f_Y(y)$ in the form

$$f_Y(y) = \frac{1}{\sqrt{2}\,\Gamma(\frac{1}{2})} e^{-\frac{1}{2}y} y^{(1/2)-1} \quad \text{for } y > 0$$

$$= 0 \quad \text{for } y \leq 0$$

This probability density function is known as the Chi-square probability density function

4d)

(i) Probability that there is more than one Call in an interval $[t, t+\Delta t)$ in $o(\Delta t)$

(ii) Probability that there is exactly one call in an interval $[t, t+\Delta t)$ is $\lambda \Delta t + o(\Delta t)$

(iii) Probability that there are K calls in an interval $[t, t+h)$ depends only on ther length h of the interval and not on t

(IV) The events that are K calls in the interval $[t_1, t_2]$ and j calls in the ~~inver~~ interval $[t_3, t_4]$ are independent for every $K \geq 0$ and $j \geq 0$ whenever $0 \leq t_1 \leq t_2 \leq t_3 < t_4 < \infty$

5. a)

$$E(Y|X=x) = \mu_y + \rho \frac{\sigma_y}{\sigma_x}(x-\mu_x)$$

$$V(Y|X=x) = \sigma_y^2(1-\rho^2)$$

b) $E(Y|X=1350) = 117 + 0.58 \times \frac{7.2}{126}(1350-980)$

$= 129.26$

$$V(Y|X=1350) = (7.2)^2(1-(0.58)^2) = 34.4$$

$$U = [Y|X=1350] \approx N(129.26, 34.4)$$

$$Z = \frac{U - 129.26}{\sqrt{34.4}} \sim N(0,1)$$

$\therefore P(Y \le 120 \mid X=1350) = P\left(Z \le \frac{120-129.26}{\sqrt{34.4}}\right)$

$= \phi(-1.58)$

$= 1 - \phi(1.58)$

$= 0.057$

(c) Joint pdf of (X,Y) is

$f_{X,Y}(x,y) = e^{-(x+y)}, \quad x>0, y>0$

$|J| = v$

Joint pdf of (U,V) is

$f_{U,V}(u,v) = f_{X,Y}(x,y)\,|J|$

$= v.e^{-v} \quad v>0, \quad 0<u<1$

Marginal pdf of U is

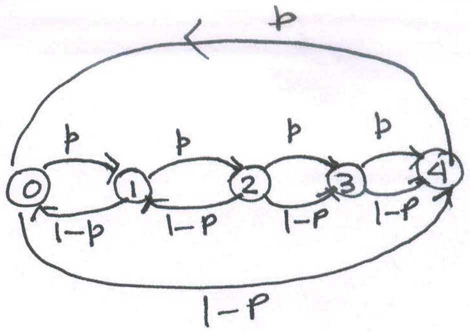
$f_U(u) = \int_0^\infty v e^{-v}\,dv = 1 \quad 0<u<1$

Marginal pdf of V is

$f_V(v) = v e^{-v} \quad v>0$

6. a)

$$P = \begin{bmatrix} 0 & p & 0 & 0 & 1-p \\ 1-p & 0 & p & 0 & 0 \\ 0 & 1-p & 0 & p & 0 \\ 0 & 0 & 1-p & 0 & p \\ p & 0 & 0 & 1-p & 0 \end{bmatrix}$$



Equivalance class $= \{0, 1, 2, 3, 4\}$

$\underset{\sim}{\pi} = \underset{\sim}{\pi} P \qquad \underset{\sim}{\pi} = (\pi_0, \pi_1, \pi_2, \pi_3, \pi_4)$

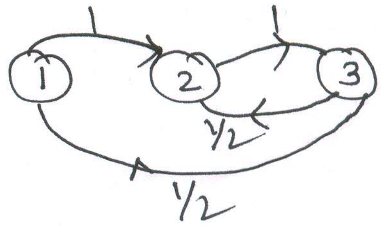
$$\Rightarrow (1-p)\pi_1 + p\pi_4 = \pi_0$$
$$p\pi_0 + (1-p)\pi_2 = \pi_1$$
$$p\pi_1 + (1-p)\pi_3 = \pi_2$$
$$p\pi_2 + (1-p)\pi_4 = \pi_3$$
$$(1-p)\pi_0 + p\pi_3 = \pi_4$$
$$\pi_0 + \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$

$\Rightarrow \pi_0 = \pi_1 = \pi_2 = \pi_3 = \pi_4 = \frac{1}{5}$

(b)

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

equivalance class $= \{B_1, B_2, B_3\}$



Since Irreducible, finite State markov chain

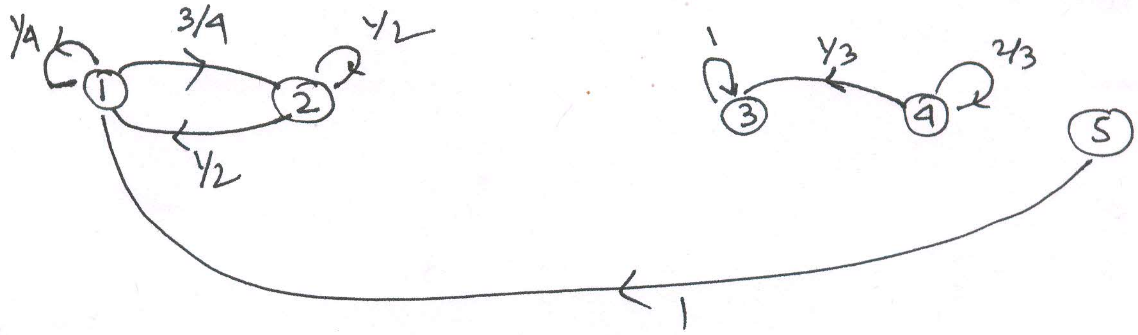
therefore all States are positive recurrent

period of State 1 is $d(1) = \gcd\{3, 5, 7, \ldots\} = 1$

" " " 2 is $d(2) = \gcd(2, 3) = 1$ aperiodic

" " " 3 " $d(3) = \gcd\{2, 3\} = 1$

6 c)



equivalance class $\{1,2\}, \{3\}, \{4\}, \{5\}$

$$f_1 = f_1^{(1)} + f_1^{(2)} + \cdots = \frac{1}{4} + \frac{3}{4} \times \frac{1}{2} + \frac{3}{4} \times \left(\frac{1}{2}\right)^2 + \cdots$$

$$= \frac{1}{4} + \frac{3}{4} = 1$$ State 1 recurrant

$1 \leftrightarrow 2$ $\therefore$ State 2 recurrant

$f_3 = 1$ State 3 recurrent

$f_4 = \frac{2}{3} < 1$ State 4 transicent

$f_5 = 0 < 1$ State 5 transient

~~Pe~~ Period of states

$d(1) = \gcd\{1, 2, 3, \cdots\} = 1$

$d(2) = 1 = d(3) = d(4)$

$d(5)$ is undefined.

7. a)

(i) Let $N(t)$ denotes the number of arrivals of Customers in $(0, t] \sim P.P(\lambda)$

$\lambda = 2$

$$P(N(10)=3) = \frac{e^{-2\times 10}(2\times 10)^3}{3!}$$

(ii) Let T denotes time between 2 Consecutive arrivals

i.e interarrival time

$T \sim exp(\lambda) \qquad \lambda = 2$

$$P(1<T<2) = P(T<2) - P(T<1)$$

$$= 1 - e^{-\lambda\times 2} - (1 - e^{-\lambda\times 1})$$

$$= e^{-2} - e^{-4}$$

$$= e^{-2}(1 - e^{-2})$$

7. b)

(i) $\lambda = 6$, $t = 7$

$$P[N(t)=5] = \frac{e^{-6\times 7}\times(6\times 7)^5}{\lfloor 5}$$

$$= 6.26\times 10^{-13}$$

(ii) Here rate is λp and $t=5$

$$\Rightarrow \lambda p t = 6\times\tfrac{1}{3}\times 5 = 10$$

$$P[N(5)=0] = e^{-10}$$

$$P[N(5)=1] = 10\times e^{-10}$$

$\therefore$ Prob of at least two particles emit with bluish light ~~is~~ in 5 min is

$$= 1 - e^{-10}(11) = 0.9995$$

(7 c)

State 0: Previous trial was a success, this trial is a success

State 1: Previous trial was failure, this trial is a success

State 2: Previous trial was success, this trial is a failure

State 3: Previous trial was failure, this trial is a failure

Transition probability matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.8 & 0 & 0.2 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 & 0.5 \end{bmatrix} \end{array} \quad \text{———— (1)}$$

$$\pi_0 = 0.8\,\pi_0 + 0.5\,\pi_1$$

$$\pi_1 = 0.5\,\pi_2 + 0.5\,\pi_3$$

$$\pi_2 = 0.2\,\pi_0 + 0.5\,\pi_1$$

$$\pi_3 = 0.5\,\pi_2 + 0.5\,\pi_3$$

$\pi_1 = \pi_3$

$\pi_0 = 2.5\,\pi_1$

$\pi_2 = \pi_3$

$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1$

$\Rightarrow 2.5\,\pi_1 + 3\,\pi_1 = 1$

$\Rightarrow \pi_1 = \dfrac{1}{5.5} = 0.1818$

$\pi_0 = 0.4545$

$\therefore$ Prob. the trials that are success

$$= \pi_0 + \frac{\pi_1}{2} + \frac{\pi_2}{2} = 0.6363$$