

Assignment - I

1. Find The Laplace transform of

a) $f(t) = \sin 2t \sin 3t$, b) $f(t) = \sin^3 2t$

c) $f(t) = \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^3$, d) $f(t) = \sin \sqrt{t}$

2. Express the following functions in terms of the Heaviside unit-step function and find their L.T.s

a) $f(t) = \begin{cases} 1, & 0 \leq t < 2 \\ t^2, & t \geq 2 \end{cases}$

b) $f(t) = \begin{cases} 0, & 0 \leq t < 3 \\ e^t, & 3 \leq t < 4 \\ 0, & t \geq 4 \end{cases}$

c) $f(t) = \begin{cases} \sin t, & 0 \leq t < \pi/4 \\ \sin t + \cos(t - \pi/4), & t \geq \pi/4 \end{cases}$

3. Find L.T. of the periodic function in $(0, \infty)$

$$f(t) = \begin{cases} \sin \omega t, & 0 < t \leq \pi/\omega \\ 0, & \pi/\omega \leq t < 2\pi/\omega \end{cases}$$

in $(0, \infty)$.

4. Find L.T. of

a) $t \sin 4at$, b) $t \sin 3t \cos 2t$

c) $\int_0^t \frac{1}{u} (\cos au - \cos bu) du$, d) $\int_0^t \frac{e^u \sin u}{u} du$

5. Find

a) $L[t^2 u(t-1) + 8(t-1)]$, c) $J_0(at)$

b) $L[t^3 \delta(t-4)]$, d) $e^{2t} \operatorname{erf}(\sqrt{t})$

e) $L[f(t)]$, $f(t) = [t]$, which is the greatest integer $\leq t$.

Assignment - II

1. Find L^{-1} of the following function:

a) $\frac{s}{s^4 + s^2 + 1}$, b) $\log \frac{s^2 + 1}{s(s+1)}$, c) $\log \left(\frac{s+1}{s-1} \right)$,

d) $\frac{s e^{-s/2} + \pi e^{-s}}{s^2 + \pi^2}$, e) $\frac{e^{-s}}{(s+1)^3}$.

f) $\cot^{-1} \frac{s}{2}$, g) $\tan^{-1} \frac{2}{s^2}$, h) $\frac{1}{1 + \sqrt{1+s}}$

i) $\frac{1}{\sqrt{s}(s-1)}$, j) $\log \frac{s+2}{s+3}$

k) $\frac{1}{s} \left[\tan^{-1} \frac{a}{s} - \frac{as}{a^2 + s^2} \right]$, l) $\frac{1}{\sqrt{s} - \sqrt{a}}$

2. Evaluate the integrals by L.T

a) $\int_0^\infty e^{-tx^2} dx$; b) $\int_0^\infty e^{-2x} \frac{\sin 3x - \sin 4x}{x} dx$;

Definite integrals.

c) $\int_0^\infty e^{-4t} \cos 2t dt$; d) $\int_0^\infty \frac{\sin^2 \theta}{\theta^2} d\theta$

e) $f(t) = \int_0^\infty \frac{\cos t\theta}{1 + \theta^2} d\theta$

f) $\int_0^\infty e^{-2t} \frac{\sin ht \sin t}{t} dt$.

3. Solve the following integral equations by L.T.

a) $y(t) = \sin 2t + \int_0^t y(\tau) \sin 2(t-\tau) d\tau$

b) $\frac{dy}{dt} + 4y + 5 \int_0^t y dt = e^{-t}$, $y(0) = 0$.

c) $f(t) = 1 - \sin ht + \int_0^t (1+\tau) y(t-\tau) d\tau$.

d) $\int_0^t f(\tau) J_0(2(t-\tau)) d\tau = \sin 2t$.

Assignment - III

1.8 Use Laplace transformation technique to solve the following IVPs

a) $\ddot{y}(t) + 2\dot{y}(t) + 2y(t) = 5u(t - 2\pi)\sin t,$

$$y(0) = 1, \dot{y}(0) = 0$$

b) $\ddot{y}(t) + 4y(t) = 10\delta(t - 3), y(0) = \dot{y}(0) = 0$

c) $\ddot{y}(t) + y(t) = f(t), y(0) = \dot{y}(0) = 0$

where $f(t) = n+1$ for $n\pi \leq t \leq (n+1)\pi,$
 $n = 1, 2, \dots$

d) $t\ddot{y}(t) + \dot{y}(t) + ty(t) = 0, y(0) = 2, \dot{y}(0) = 0$

e) $\ddot{y} + t\dot{y} - y = 0, y(0) = 0, \dot{y}(0) = 1.$

2. Solve the following BVP by L.T.

a) $y'' + 9y = \cos 2t, y(0) = 1, y(\pi/2) = -1.$

b) $y'' + 4y = 0, y(0) = 0, y(\pi/4) = -1$

3. Solve the IVP

$$y'' + ty' - y = 0, y(0) = 0, y'(0) = 1.$$

4. $\frac{dx}{dt} - y = e^t$ & $\frac{dy}{dt} + x = \sin t$

$$x(0) = 1, y(0) = 0,$$

5. $3x' + y' = 1 - 2x$

$$x' + 4y' + 3y = 0, y(0) = x(0) = 0$$

6. Find $y(t)$ when

$$y(t) = 1 - \sin ht + \int_0^t (1 + \tau)y(t - \tau)d\tau$$

7. Show that $\int_0^t J_0(\tau)J_0(t - \tau)d\tau = \sin t.$