An Analytical Formulation of Pure Shear Boundary Condition for Assessing the Response of Some Typical Sites in Mumbai

Raj Banerjee, Aniruddha Sengupta

***Abstract*—**An earthquake event, associated with a typical fault rupture, initiates at the source, propagates through a rock or soil medium and finally daylights at a surface which might be a populous city. The detrimental effects of an earthquake are often quantified in terms of the responses of superstructures resting on the soil. Hence, there is a need for the estimation of amplification of the bedrock motions due to the influence of local site conditions. In the present study, field borehole log data of Mangalwadi and Walkeswar sites in Mumbai city are considered. The data consists of variation of SPT N-value with the depth of soil. A correlation between shear wave velocity (V_s) and SPT N-value for various soil profiles of Mumbai city has been developed using various existing correlations which is used further for site response analysis. A MATLAB program is developed for studying the ground response by performing two- dimensional linear and equivalent linear analyses for some of the typical Mumbai soil sites using pure shear (Multi Point Constraint) boundary condition. The model is validated in linear elastic and equivalent linear domain using the popular commercial program, DEEPSOIL. Three actual earthquake motions are selected based on their frequency contents and durations and scaled to a PGA of 0.16g for the present ground response analyses. The results are presented in terms of peak acceleration time history with depth, peak shear strain time history with depth, Fourier amplitude versus frequency, response spectrum at the surface, etc. The peak ground acceleration amplification factors are found to be about 2.870, 3.750 and 2.691 for Mangalwadi site and 3.39, 3.42 and 3.75 for Walkeswar site using 1979 Imperial valley Earthquake, 1989 Loma Gilroy Earthquake and 1987 Whitter Narrows Earthquake, respectively. In the absence of any site-specific response spectrum for the chosen sites in Mumbai, the generated spectrum at the surface may be utilized for the design of any superstructure at these locations.

***Keywords*—**DEEPSOIL; Ground response analysis; Multi Point Constraint; Response spectrum.

I. INTRODUCTION

The area considered in this work belongs to the commercial capital of India, Mumbai. The city of Mumbai has its center at 19.0760° N, 72.8777° E and is sprawling in an area of approximately 603.4 km^2. The city has a population of approximately 20.7 million. Owing to its high population density, it is essential to limit the possibility of severe damages to the existing buildings during strong and moderate earthquakes. Mumbai is located in Seismic Zone III as per IS:1893-2002 [1] signifying that the city may be subjected to intensity VII damage as per MSK64 intensity scale. a region with moderate seismic hazard. The region is characterized by low to moderate level of seismic activity [2]. Some of the large and damaging earthquakes, such as, Koyna (1967), Killari (1993), Jabalpur (1997), and Kachchh (2001) earthquakes have occurred in the region. Mumbai is located near the Panvel seismic source zone, which is known to be seismically active [3], [4]. Therefore, seismic ground response analysis is required to develop the site-specific response spectrum for the design of superstructures in the region. The one-dimensional ground response analysis is commonly used method to estimate the ground response under earthquake excitation. In this study, 2-D plane strain site response analysis has been performed for two sites (Mangalwadi and Walkeswar) of Mumbai city in both linear and equivalent linear domains for different levels of ground shaking (i.e., for different frequency content, duration, etc.). A MATLAB [5] code with lumped mass model and pure shear boundary conditions, and performing time-domain integration of equations of motion step-by-step have been developed for the purpose. It assumes that the soil layers are horizontal and the response of the soil site is predominantly due to the horizontally-polarized shear waves that propagate vertically from the underlying bedrock. The linear approach of the ground response analysis is performed with a constant value of shear modulus (G) and damping ratio (D) for the induced level of shear strain in each soil layer. However, the behaviour of the soil is inelastic and its material properties vary spatially. In the equivalent linear approach, an iterative procedure is used to obtain the values of the shear modulus and the damping compatible with the representative effective shear strain in each soil layer. Though equivalent linear methods are fast and provides reasonable estimates for most of the practical problems, it is an approximate solution to the actual non-linear process of seismic ground response.

Raj Banerjee is with the Bhabha Atomic Research Center, Mumbai, 400485 India (phone: 02225593548; e-mail: rbanerjee@barc.gov.in).

Aniruddha Sengupta is with the Civil Engineering Department, Indian Institute of Technology, Kharagpur, 721302 India (e-mail: sengupta@civil.iitkgp.ernet.in).

II. ANALYSIS METHODOLOGY AND MODEL VALIDATION

A MATLAB [5] program is developed for studying the ground response analysis by performing two dimensional linear and equivalent linear analysis in time domain, for some of the typical Mumbai soil sites using pure shear (Multipoint constraint [6] boundary condition. This type of boundary condition is used to simulate pure shear type of movement in a soil column in which the unwanted reflections from the boundaries are minimized to a significant extent in comparison to the absorbing boundaries. In equivalent linear analysis, an iterative approach is followed [7], in which, the initial estimates of the values of G_i and D_i, corresponding to small strains, are made for each soil layer. The estimated G_i and D_i are used to compute the ground response, including the time histories of shear strain for each layer. The effective shear strain in each layer is determined from the maximum shear strain in the computed shear strain time history. From this effective shear strain, new equivalent linear values, $G_{i+1}$and $D_{i+1}$are chosen for the next iteration. The above steps are repeated until the difference between the previous and new values is less than 5-10%. The iteration converges within 3 to 4 steps normally [8]. In both, linear and equivalent linear approach, equations of motion are solved in discrete time increments using time domain analysis. The following dynamic equation of equilibrium is solved:

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = [M][I]\{\ddot{u_g}(t)\} \quad (1)$$

where, [M] is the lumped mass matrix, [K] is the stiffness matrix, [I] is the influence matrix (=1 in the direction of the application of motion, and 0 in the direction, where no motion is applied) and [C] is the damping matrix of the soil. The dynamic equilibrium equation, Eq. (1), is solved numerically at each time step using the Constant Average Acceleration method [9]. The base of the soil column is modeled as infinitely stiff. For the i^{th} layer of the soil, the mass is lumped at each node of an 8-noded, 2-D quadrilateral element (= ρV/8, where ρ is the density of soil, V is the volume of the element). The formulation of the stiffness matrix requires the following basic definition:

$$[K] = \iint [B]^T[D][B]dv \quad (2)$$

which is modified in isoparametric formulation, as (η = y/a, ξ=x/a where 'a' is the half of the element size):

$$[K] = t\iint_{-1}^{1} [B]^T[D][B]|J|d\xi d\eta \quad (3)$$

Where, 't' is the out of plane thickness of the element, [D] is the constitutive matrix in plane strain and given by:

$$[D] = \frac{E}{(1+\vartheta)(1-2\vartheta)}\begin{bmatrix} 1-\vartheta & \vartheta & 0 \\ \vartheta & 1-\vartheta & 0 \\ 0 & 0 & \frac{(1-2\vartheta)}{2} \end{bmatrix} \quad (4)$$

in which, E is the elastic modulus and ϑ is the Poisson's ratio. For the formulation of B-matrix, the following shape functions for a 8-noded element are defined:

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)(-\xi-\eta-1)$$
$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1)$$
$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1)$$
$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)(-\xi+\eta-1)$$
$$N_5 = \frac{1}{4}(1-\xi)(1-\eta)(1+\xi)$$
$$N_6 = \frac{1}{4}(1-\xi)(1-\eta)(1+\eta)$$
$$N_7 = \frac{1}{4}(1-\xi)(1+\eta)(1+\xi)$$
$$N_8 = \frac{1}{4}(1-\xi)(1-\eta)(1+\eta) \quad (5)$$

The node numbering in a single element is given in Fig. 1:

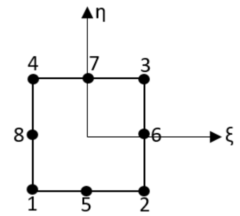


Fig. 1 Node numbering in a 8-noded quadratic quadrilateral element.

The [C] matrix is a combination of mass and stiffness matrices and is of the form [10]:

$$[C] = \alpha_R[M] + \beta_R[K] \quad (6)$$

The values of α_R and β_R are calculated by considering the first and third natural frequencies of the soil column. For an elastic analysis, the damping ratio is assumed to be 5%. In the equivalent linear analysis, the damping ratio is calculated by the variation of shear strain with damping ratio for a soil. For a frequency independent damping ratio, the formulation of layered damping for a multi-layered soil is followed as per [11]. The boundary conditions are implemented in such a fashion that there is a relationship between two or more degrees of freedom (multipoint constraint). The soil column is modelled using 2-D, plane strain, 8-noded quadratic quadrilateral element with two degrees of freedom (horizontal and vertical displacements) at each node. A rigid element imposes a multipoint constraint. A rigid element is a bar element with a single degree of freedom in each node in which the axial stiffness (AE/L=w) is taken a high value. It is actually connected to the two nodes of the lateral boundaries and the axial stiffness is made high so that there is negligible axial strain in the bar element which implies that the deflection of the end nodes are same. One node acts as the "master" and another node acts as a "slave" which follows the master. The constraint equation that ties the horizontal degree of freedom is written in the following form [12]:

$$[B][u] = [A] \quad (7)$$

Where, B and A are constants. For homogeneous constraints, the value of [A] is equal to zero. The equation can be written in a modified format as:

$$[Q] = [B][u] - [A] \quad (8)$$

[Q]=0 implies the satisfaction of the constraints. "Penalty augmentation" [12] is used for implementation of the constraints at the boundary degrees of freedom. Each multipoint constraint is viewed as the presence of a fictitious elastic structural element called penalty element (w) that enforces it approximately. This element is parametrized by a numerical weight. The multipoint constraints are imposed by modifying the final assembled stiffness matrix which is submitted to the equation solver as:

$$[K_{modified}] = [K] + [B]^T w[B] \quad (9)$$

If w =0, then the constraints are ignored, hence the selection of the appropriate weights are necessary to minimize ill-conditioned solution (with respect to inversion of the stiffness matrix) as well as to avoid mesh locking. For instance, if we choose $u_{4x} = u_{8x}$, then it can be written as $u_{4x} - u_{8x} = 0$, which is a homogeneous constraint. It can be written in matrix form:

$$[1 \ -1]\begin{bmatrix} u_{4x} \\ u_{8x} \end{bmatrix} = 0 \tag{10}$$

Where, $[B] = [1 \ -1]$ and $[B]^T w[B] = w\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Hence the above matrix is incorporated into the assembled stiffness matrix at the appropriate locations of the degree of freedom. It generally implies the addition of an axially rigid bar element with axial stiffness (w) to the two tied nodes. The trade-off value of weights is difficult to find, which will encompass all the above problems. Hence a rule is followed in which the weights are chosen in this study, typically on the order of 10^6 to 10^7 for double precision (64-bit processor) to avoid numerical difficulty [12]. For validation of the model in elastic (or linear) and equivalent linear analysis, a soil profile is chosen from bore log data near Trombay in Mumbai. The data consists of variation of SPT N-value with the depth of the soil. It is a common practice to obtain shear wave velocity (V_s in m/s) using the correlation with field standard penetration test (SPT) N-values in absence of sophisticated dynamic field test data. Hence, in this paper, a modified approach has been proposed for Mumbai city to obtain shear wave velocity profile using regression analyses. This is done using the empirical correlations between the shear wave velocity and the SPT N-values for specific and all soil types, proposed by previous researchers worldwide as tabulated below:

TABLE I
CORRELATIONS OF SPT N-VALUE WITH THE SHEAR WAVE VELOCITY OF SOIL.

Correlation	Authors
$V_s=76N^{0.33}$	[13]
$V_s=82N^{0.39}$	[14]
$V_s=91N^{0.337}$	[15]
$V_s=90N^{0.34}$	[16]
$V_s=100.5N^{0.329}$	[17]
$V_s=107.6N^{0.36}$	[18]
$V_s=116.1(N+0.3185)^{0.202}$	[19]
$V_s=82.6N^{0.43}$	[20]
$V_s=95.64N^{0.301}$	[21]

The average value of these calculated shear wave velocities is used in the present study. A nonlinear regression analysis employing power model has been implemented using the measured SPT N-values with average shear wave velocity (Vs) for various soil types of Mumbai city as shown in Fig. 2.

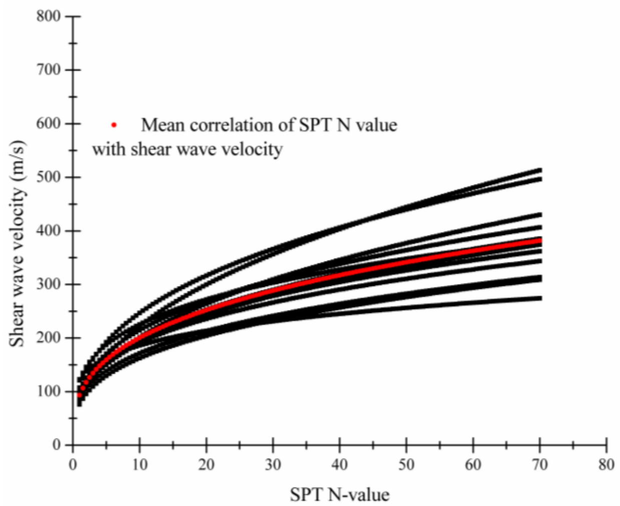


Fig. 2 Variation of shear wave velocity of soil with N-value for Mumbai city

The obtained best fit relationship of shear wave velocity with SPT N-value for the city of Mumbai is given by:

$$V_s\left(in\,\frac{m}{s}\right) = 93.34(N)^{0.33162} \tag{11}$$

Using the above correlations, the shear wave velocity with depth for the considered soil profile in Mumbai is shown in Table II. The soil up to a depth of 6m is yellowish sandy silty soil. The bedrock is found below 6m. The schematic diagram of the soil column along with the layer depths is shown in Fig. 3.

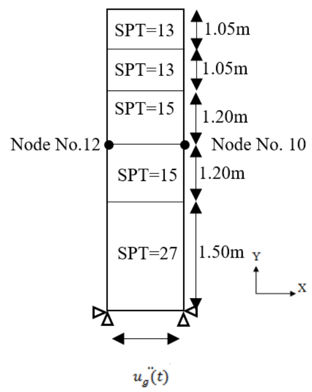


Fig. 3 Schematic diagram of soil column for Trombay in Mumbai city.

The validation of the model is shown in terms of acceleration variation with depth and maximum shear strain with depth in both linear and equivalent linear analysis for 1999 Chi Chi Earthquake, which is shown in Figs. 4 and 5 using the MATLAB [5] program and DEEPSOIL v 5.1 [22].

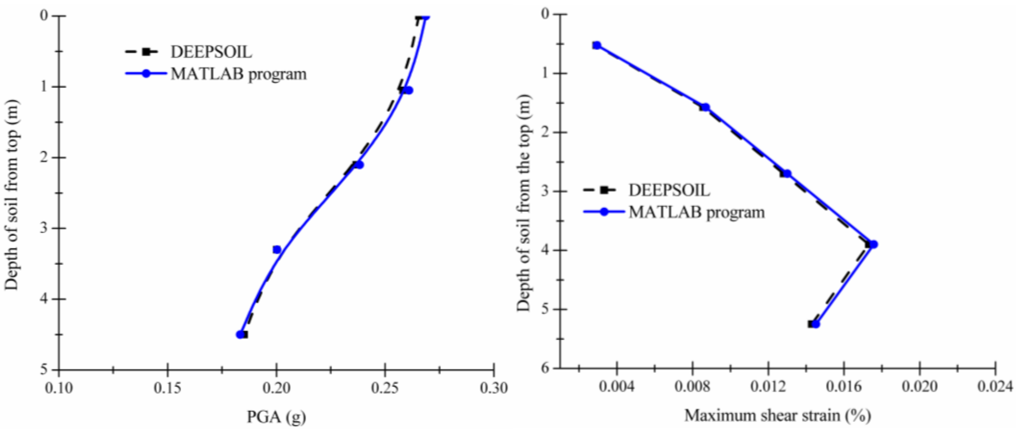


Fig. 4 Peak acceleration and maximum shear strain with depth for Trombay site in elastic domain.

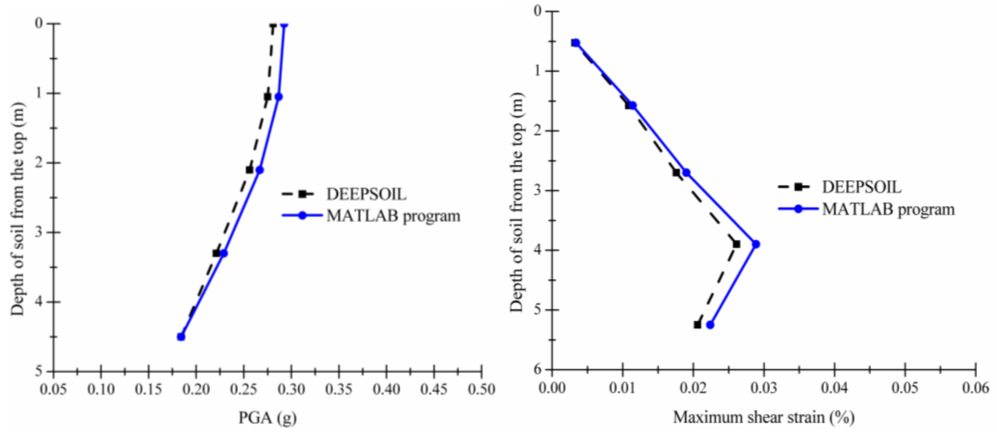


Fig. 5 Peak acceleration and maximum shear strain with depth for Trombay site in equivalent linear domain.

In the "equivalent linear" site response analysis, the strain compatible shear modulus (G) and damping ratio (D) values are utilized to model the nonlinear behavior of the soil.

TABLE II
SHEAR WAVE VELOCITY OF SOIL WITH DEPTH

Depth of soil	SPT N-value	Shear wave velocity (m/s)
(0.0-2.1)m	13	218.5112
(2.1-4.5)m	15	229.1306
(4.5-6.0)m	27	278.4432

Soil is a complex material which exhibits nonlinear behavior even at a small strain level. As the strain is increasing, the G value reduces and the D value increases, suggesting both modulus (G/G_{max}) and D are function of shear strain. The G/G_{max} and damping ratio curves are different for different soil types and are usually obtained from the laboratory dynamic tests on the given soil. However, due to lack of such facilities available on regional level, the use of standard material curves for each type of soil is widely practiced. Like, if the soil is sandy in nature, the G/G_{max} and the D curves for sand, developed by [23], are used due to the absence of site specific data on the shakedown strength and damping for a sandy soil. In equivalent linear analysis, the maximum frequency considered for the input motions is up to the Nyquist frequency. During equivalent linear analysis, while due consideration is given to amplitude, no to very limited information about the frequency content of the input motion to be considered in ground response analysis is available. In SHAKE 2000 [8], it is a frequency based analysis tool having default frequency set to 15 Hz which means that the analysis neglects frequency beyond 15 Hz of the input motion. The effects of maximum frequency to be considered in the input motion, which affects the response of the ground motion at a site is shown by [24]. Thus, in the present study, the code has been developed which considers the frequency content of the input motion (up to Nyquist frequency (the maximum frequency content of the ground motion which is calculated by the expression, $f_{max} = \frac{1}{2\Delta t}$) where Δt is the time interval of the ground motion), thus giving due importance to all the frequencies present in the input motion.

For proper working of the Multipoint constraint, it is to be checked that the master and slave node follows the same deformation at any instant of time which means that if we take the difference of the movement between the two nodes, it should be negligible which is shown in Fig. 6 in which the differences between the horizontal displacements for node 10 and 12 is given below,

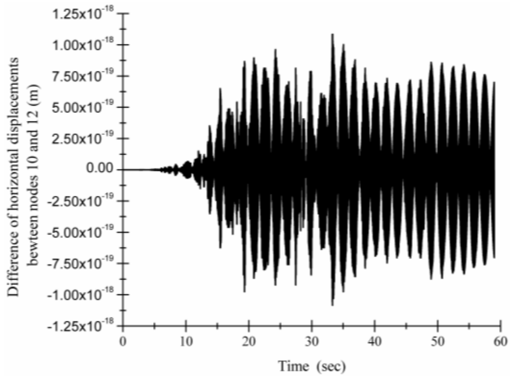


Fig. 6 Difference between the horizontal displacements at Nodes 10 and 12

The figure shows insignificant difference between the movements of the two nodes. This implies that the two nodes are in unison during the dynamic loading. This validates the working of the boundary condition as well as the analysis of the soil column in both, linear elastic and equivalent linear, domains. It is seen that the results of the MATLAB [5] program matches closely with the results of the DEEPSOIL v 5.1 [22] in both domains, with the degree of deviation within the limits of acceptance which validates the program with tie boundary conditions.

III. Site Response Analysis of Mangalwadi and Walkeswar Sites in Mumbai City

The field borehole data from Mangalwadi site (MBH-1) and Walkeswar site (WBH-1) in Mumbai have been used for the present analyses. The ground water tables for Mangalwadi and Walkeswar sites have been considered 4m below the ground level (GL). The bore log details for the above sites have been presented in Tables III and IV.

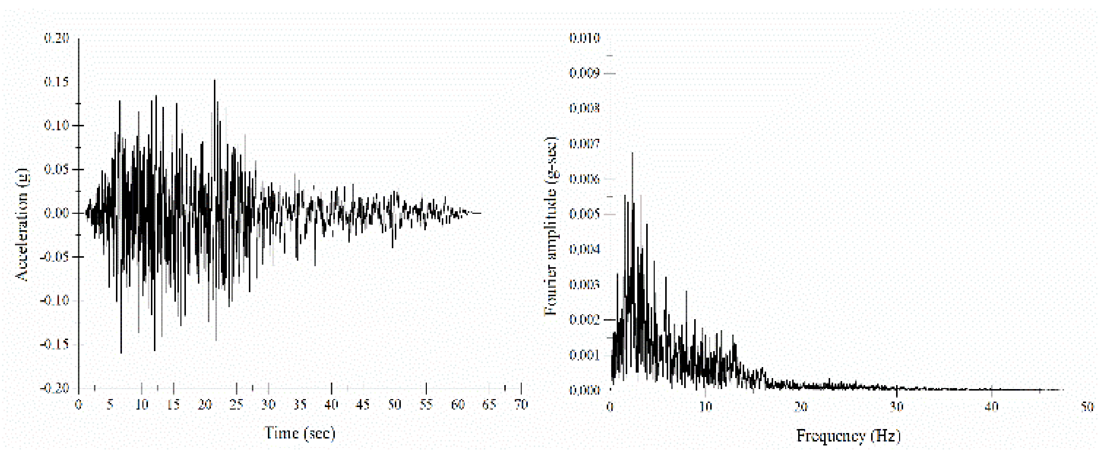
TABLE III
PROPERTIES OF SOIL FOR MANGALWADI (MBH-1) SITE.

Type of soil	Depth of soil	SPT N-value	Shear wave velocity (m/s)
Fill	(0.0-1.5)m	10	200.3032
Loose sand	(1.5-3.0)m	12	212.7874
Loose sand	(3.0-4.5)m	13	218.5112
Loose sand	(4.5-6.0)m	16	234.0874
Black clay	(6.0-8.0)m	20	252.0666
Yellowish clay	(8.0-9.8)m	25	271.4268

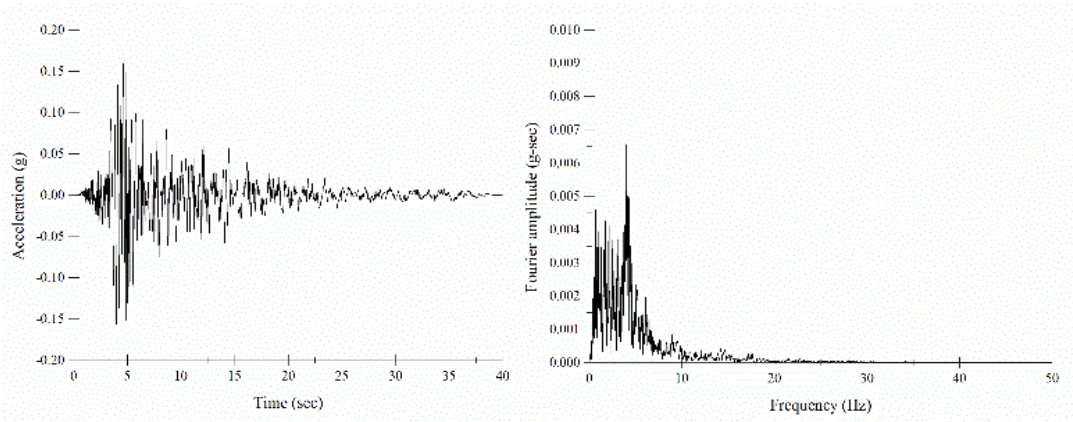
TABLE IV
PROPERTIES OF SOIL FOR WALKESWAR (WBH-1) SITE.

Type of soil	Depth of soil	SPT N-value	Shear wave velocity (m/s)
Fill	(0.0-3.5)m	11	206.7352
Silty gravel	(3.5-5.0)m	19	247.8173
Silty gravel	(5.0-7.0)m	22	260.1609
Silty gravel	(7.0-10.5)m	62	366.8265

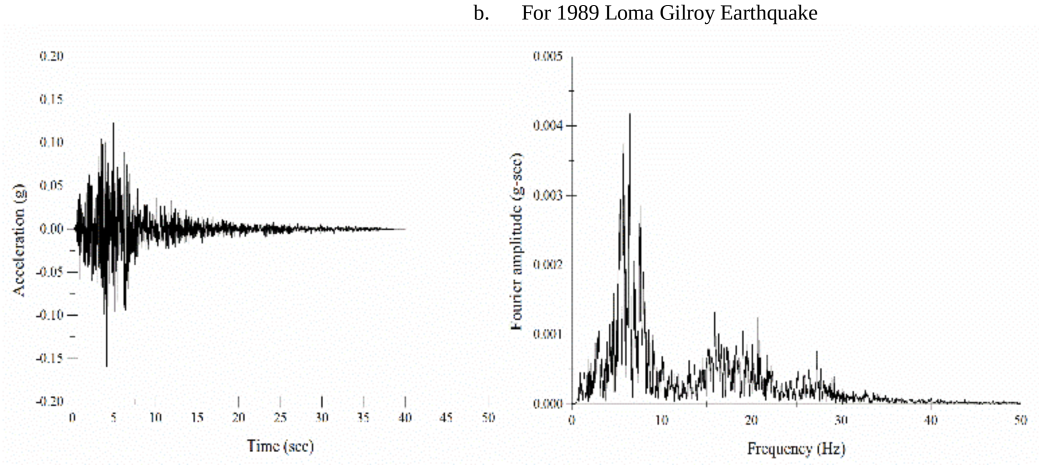
The bedrock is considered as rigid and hence energy dissipation due to the reflection of seismic waves at the bedrock/soil boundaries is not considered. In the present study, three actual earthquake motions, 1979 Imperial Valley Earthquake, 1989 Loma Gilroy Earthquake and 1987 Whitter Narrows Earthquake, are selected based on their frequency contents and durations. As the city of Mumbai lies in Zone-III as per IS 1893:2002, the PGA for this zone is 0.16g, hence all the motions have been scaled to PGA of 0.16g. The records of the strong motions are obtained from DEEPSOIL v 5.1 [22]. The earthquake characteristics of these motions, like, predominant period (from smoothened Fourier spectrum) and significant duration (from Arias intensity) are obtained. The selected motions have predominant time period of 0.320s, 0.168s and 0.160s, respectively. Their corresponding significant durations are 29.70s, 12.615s and 8.325s, respectively, which represents the wide variation of the ground motion parameters. The time histories and the Fourier transform of 1979 Imperial Valley Earthquake, 1989 Loma Gilroy Earthquake and 1987 Whitter Narrows Earthquake are shown in Fig. 7.



a. For 1979 Imperial Valley Earthquake



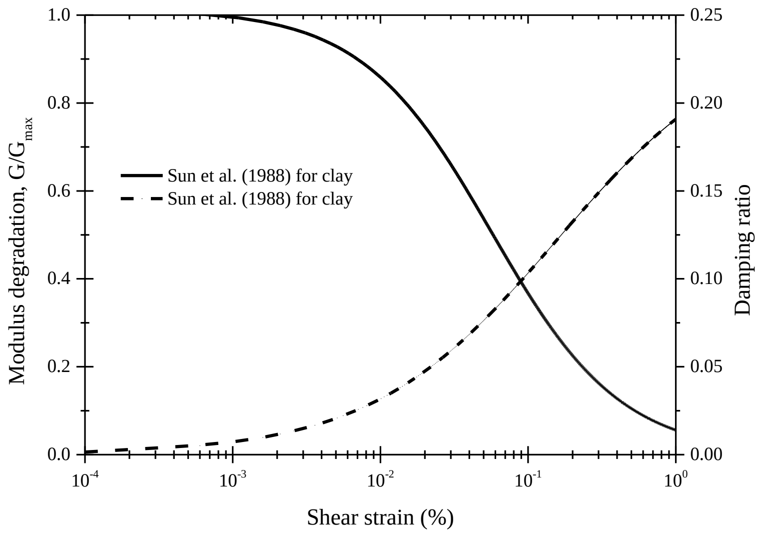
b. For 1989 Loma Gilroy Earthquake



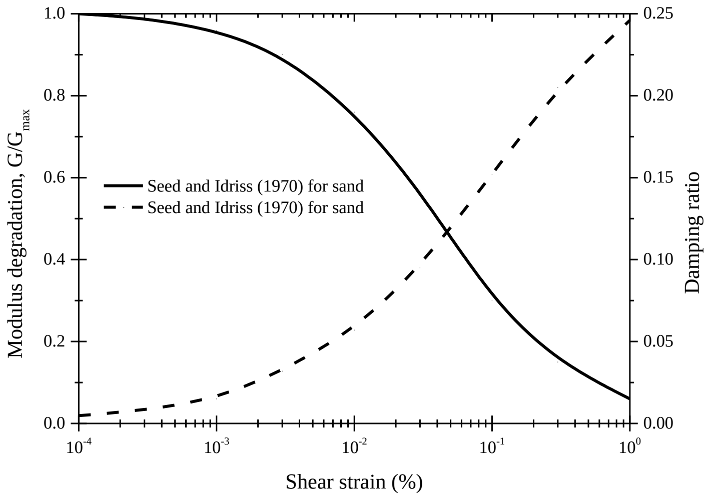
c. 1987 Whitter Narrows Earthquake

Fig. 7 Selected earthquake motions in time and frequency domains.

For sand, and clay layers, the average G/G_{max} and D curves developed by [23] and [25] are used which are shown in Fig. 8 below:



a. For clay

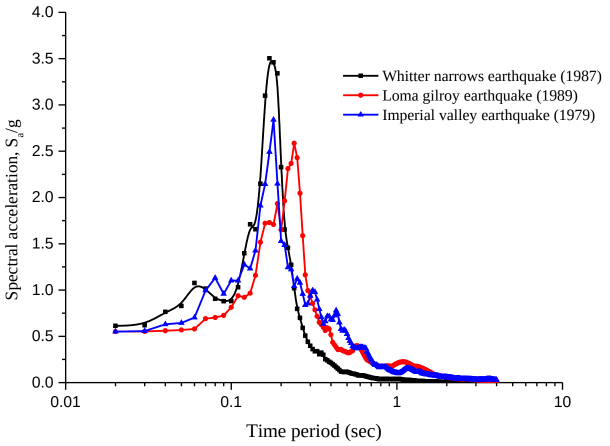


b. For sand

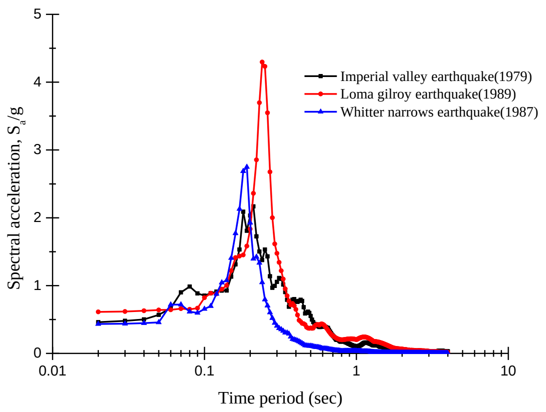
Fig. 8 G/G_{max} and damping ratio curves for sand and clay

IV. RESULTS AND DISCUSSION

The acceleration response spectrum at ground level for Mangalwadi and Walkeswar sites for the three selected motions are evaluated and the results are shown in Fig. 9 for 5% damping ratio of the soils.



a. For Walkeswar site



b. For Mangalwadi site

Fig. 9 Response spectrum at the top surface of soil at Walkeswar and Mangalwadi sites for the selected ground motions.

It is observed that the amplification of motions for the Imperial Valley, Loma Gilroy and Whitter Narrows earthquakes is about 2.870, 3.750 and 2.691 at the Mangalwadi site. They are 3.39, 3.42 and 3.75 at the Walkeswar site. The variations of acceleration with depth at both the sites are shown in Fig. 10.

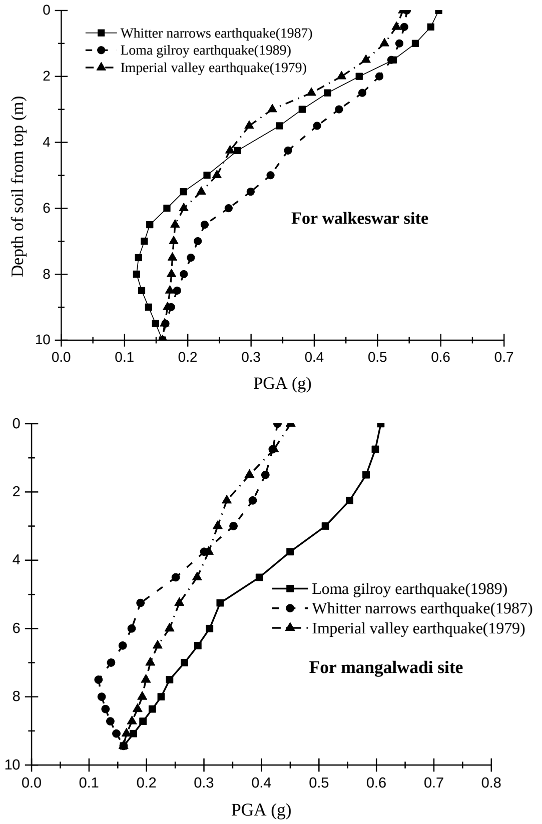
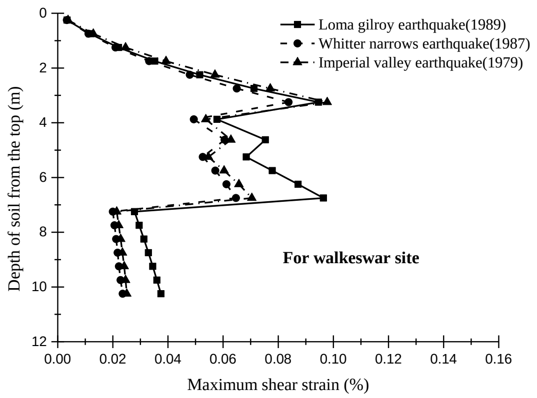


Fig. 10 Variation of acceleration with depth at Walkeswar and Mangalwadi sites.

At Mangalwadi site, the peak spectral acceleration is about 4.29g in case of Loma Gilroy motion and is observed at 0.24s. The peak spectral accelerations for Imperial Valley and Whitter Narrows motions are 2.16g and 2.74g at 0.18s and 0.21s, respectively. At Walkeswar site, the peak spectral acceleration is about 3.50g at 0.18s in case of Whitter

Narrows motion. The peak spectral accelerations for Imperial Valley and Whitter Narrows motions are 2.75g and 2.60g at 0.18s and 0.24s, respectively. The distribution of peak shear strain with depth at Mangalwadi and Walkeswar sites is shown in Fig. 11. The maximum strain at Mangalwadi site is found to be around 0.18% for Loma Gilroy motion and occurring at a depth of 6m from the top surface. The maximum strain experienced at Walkeswar site is around 0.10% and occurring at a depth of 7m from the top surface of soil.



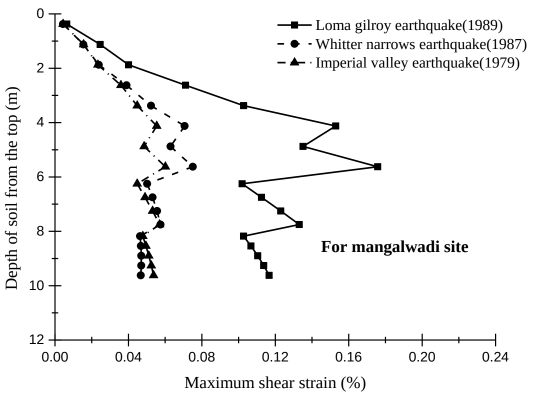


Fig. 11 Variation of maximum shear strain in soil with depth at Walkeswar and Mangalwadi sites

It is observed that the maximum strain at Mangalwadi site is around 1.80 times the strain at Walkeswar site although they are predicted at different depths of soil. It is also observed that wherever there is a change in layer properties, there is an abrupt change in the shear strain in the soil which is observed in Fig. 11.

V. Conclusion

The ground response analyses of two typical Mumbai city soil sites have been presented for three strong motion earthquakes. It is observed that the local soil characteristics at the two sites have a profound influence on the ground responses. The natural frequency of soil for Mangalwadi site is 5.89 Hz, whereas the natural frequency of soil for Walkeswar site is 6.22 Hz. The selected motions have predominant time period of 0.320s, 0.168s and 0.160s. Hence, the predominant frequency of Loma Gilroy motion is 5.95 Hz which is close to the natural frequency of Mangalwadi site, hence this earthquake gives the maximum response as it can be seen from the plot of PGA. Hence, it explains the amplification of motions for the Imperial Valley, Loma Gilroy and Whitter Narrows earthquakes which is found to be around 2.870, 3.750 and 2.691. For Walkeswar site, the predominant frequency of Whitter narrows motion is 6.25 Hz which is close to the natural frequency of Walkeswar site, hence this earthquake gives the maximum response as it can be seen from the plot of PGA. Hence, it explains the amplification of motions for the Imperial Valley, Loma Gilroy and Whitter Narrows earthquakes which is found to be around 3.39, 3.42 and 3.75 respectively. Hence, it is concluded that as the predominant frequency reaches close to the fundamental frequency of the soil column, the amplification of ground motion increases by a significant amount which is in the range of 3.75-3.80 times. The peak ground acceleration amplification factors for 1989 Loma Gilroy motions are found to be about 3.793 at the Mangalwadi site and 3.21 at the Walkeswar site. The site specific response spectra at the two sites have been evaluated for three strong motion earthquakes. These response spectra may be readily used by engineers for the seismic analyses and design of any superstructures in the city of Mumbai.

References

[1] IS 1893 (Part 1): *Indian Standard Criteria for Earthquake Resistant Design of Structures, Part1—General Provisions and Buildings (Fifth Revision)*, Bureau of Indian Standards, New Delhi, 2002.

[2] K. Jaiswal and R. Sinha, "Probabilistic Seismic Hazard Estimation for Peninsular India," *Bull. Seism. Soc.Am,* vol. 97, no. 1B, pp. 318-330, 2007.

[3] D.R. Nandy, "Neotectonism and Seismic Hazards in India," *Indian J. Geol,* vol. 67, pp. 34-48, 1995.

[4] A.G. Dessai, H. Bertrand, "The Panvel Flexure along the Western Indian Continental Margin: An Extensional Fault Structure Related to Deccan Magmatism," *Tectonophysics,* vol. 241, pp. 165-178, 1995.

[5] MATLAB and Statistics Toolbox Release, *The MathWorks, Inc.,* Natick, Massachusetts, United States, 2015b.

[6] J. Abel and M. S. Shephard, "An algorithm for multipoint constraints in finite element analysis," *International Journal for Numerical Methods in Engineering,* vol. 14, no. 3, pp. 464 – 467, 1979.

[7] S.L. Kramer, *Geotechnical earthquake engineering*. Prentice hall, 2005.

[8] P.B. Schnabel, J. Lysmer, H.B. Seed, *SHAKE—A computer program for earthquake response analysis of horizontally layered sites*, EERC report, Earthquake engineering, pp. 72-112, 1972.

[9] Chang, "Numerical characteristics of constant average acceleration method in solution of nonlinear systems," *Journal of the Chinese Institute of Engineers,* vol. 32, no. 4, pp. 519-529, 2009.

[10] J.W.S. Rayleigh, R.B. Lindsay, *The theory of sound*. 1st American ed. New York, Dover Publications, 1945.

[11] Y.M.A. Hashash, D. Park, "Viscous damping formulation and high frequency motion propagation in non-linear site response analysis," *Soil Dynamics and Earthquake Engg,* vol. 22, pp. 611–624, 2002.

[12] R. Cook, D.S. Malkus, M.E. Plesha, *Concepts and Applications of finite element analysis.* John Wiley and Sons, 1989.

[13] T. Imai, Y. Yoshimura, "Elastic wave velocity and soil properties in soft soil (in Japanese)," *Tsuchito- Kiso,* vol. 18, no. 1, pp. 17–22, 1970.

[14] Y. Ohsaki, R. Iwasaki, "On dynamic shear moduli and Poisson's ratio of soil deposits," *Soils Found,* vol. 13, no. 4, pp. 61–73, 1973.

[15] T, Imai, "P- and S-wave velocities of the ground in Japan," *In: Proceedings of the IX international conference on soil mechanics and foundation engineering* vol. 2, 1994, pp. 127–132

[16] T. Imai, Y. Yoshimura, "Elastic wave velocity and soil properties in soft soil (in Japanese)," *Tsuchito- Kiso,* vol. 18, no. 1, pp. 17–22, 1970.

[17] D.E. Sykora, and K.H. Stokoe, "Correlations of in-situ measurements in sands of shear wave velocity," *Soil Dynamics and Earthquake Engineering,* vol. 20, no. 1,pp. 125–136, 1983.

[18] G.A. Athanasopoulos, "Empirical correlations V_{so}–N_{SPT} for soils of Greece: a comparative study of reliability," *In: Proceedings of 7th international conference on soil dynamics and earthquake engineering, Computation Mechanics Publications*, Southampton, Boston, 1995, pp. 19–25.

[19] Z. Jinan, "Correlation between seismic wave velocity and the number of blow of SPT and depth," *Selected papers from the Chinese Journal of Geotechnical Engineering ASCE*, pp. 92–100, 1987.

[20] C. Hanumanthrao, G.V. Ramana, "Dynamic soil properties for microzonation of Delhi, India," *J Earth Syst Sci*, vol. 117, no. S2, pp. 719–730, 2008.

[21] R.U. Maheshwari, A. Boominathan, G.R. Dodagoudar, "Use of surface waves in statistical correlations of shear wave velocity and penetration resistance of Chennai soils," *Geotech Geol Eng*, vol. 28, pp. 119–137, 2010.

[22] Y.M.A. Hashash, D.R. Groholski, C.A. Phillips, D. Park, and M. Musgrove, "DEEPSOIL 5.1, User Manual and Tutorial,", 2012.

[23] H.B. Seed, I.M. Idriss, *Soil moduli and damping factors for dynamic response analysis*, no. EERC 70-10. University of California, Berkeley, 1970.

[24] J.K. Mondal, A. Kumar, "Impact of Higher Frequency Content of Input Motion Upon Equivalent Linear Site Response Analysis for the Study Area of Delhi," *Geotech Geol Eng*, vol. 35, pp. 959–981, 2017.

[25] J.I. Sun, R. Golesorkhi, H.B. Seed, *Dynamic moduli and damping ratios for cohesive soils,* no. EERC 88-15, University of California, Berkeley, 1988.