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# A METHOD FOR FINE TUNING THE MEMBERSHIP GRADES ASSIGNED BY EXPERTS: AN APPLICATION TO BURR HEIGHT ESTIMATION IN DRILLING

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## ABSTRACT

Design of membership grades for a fuzzy set based inference system is an important issue. Considering that the knowledge of an expert is available for the initial estimates of a fuzzy parameter, a methodology is proposed for fine tuning these estimates to enhance the performance of a fuzzy set based system. The proposed methodology combines the best of an expert's knowledge and available experimental data to predict the membership grades of fuzzy parameters. Criteria considered for the optimal membership grades are the accuracy of solution and minimum violation of expert's opinion. The proposed methodology is applied in the estimation of burr height in drilling holes. It is observed that the fine tuned values of membership grades for fuzzy input parameters give better matching of predicted and observed burr height than the initial membership grades provided by expert. Fine tuning of the initial expert's estimates enhances the performance of the burr height prediction. The methodology is suitable where limited information is available initially and information value keeps on increasing.

**Keywords:** Membership grade, Fuzzy set, Expert's knowledge, Burr height, Drilling

## 1. INTRODUCTION

Design of fuzzy membership functions greatly affects a fuzzy set based inference system. For a fuzzy input or output variable, membership grades are assigned to map numeric data to linguistic fuzzy terms. Most of the time, different estimates of a fuzzy variable are decided based on expert's opinion. However, there is a need to optimize these estimates to enhance performance.

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Different methods have been proposed in the literature for automatic generation of membership grades/membership functions. Some methods eliminate the need for an expert's opinion and knowledge is acquired from training examples. Medasani et al. (1998) provided an overview of various techniques used for membership function generation for pattern recognition. The authors are of the view that there is no single best method that can be used for all applications. Choice of a method depends on the problem at hand. Hong and Lee (1996) proposed a methodology for automatic generation of membership functions for developing a fuzzy expert system for calculating insurance fees. The output values are grouped based on the similarity between two adjacent data and triangular membership functions are used for both input and output variables. Chen and Wang (1999) stressed the fact that the parameter identification, *i.e.* deciding the number of membership functions, centre, width and cross-over slope (value of the parameter at which membership grade is 0.5) is an important step in designing fuzzy logic based systems. Furukawa and Yamakawa (1995) proposed two algorithms for pattern recognition of hand written characters. Initially, the pattern samples are classified based on their outlines and a fuzzy neuron is assigned to each class. Membership function of each fuzzy neuron is decided from the example based learning.

The neural networks have been used to generate and optimize membership functions. Yang and Bose (2006) proposed a strategy to generate membership functions for pattern recognition using self-organizing feature map technique based on unsupervised learning. Choi and Rhee (2009) proposed three different algorithms based on heuristics, histograms and fuzzy *c*-means clustering for generation of fuzzy membership functions for pattern recognition. Medaglia et al. (2002) proposed a method for generation of membership functions based on Bezier curves. An expert can provide membership grades for each point in the domain and a smooth curve can be obtained by minimizing the sum of the squared errors between the fitted membership function and data. Bai and Chen (2008) developed a methodology for the construction of membership functions for lenient-type, strict-type and normal-type grades for students' evaluation. Evolutionary algorithms have been used for the optimization of membership functions (Arslan and Kaya, 2001; Garibaldi and Ifeachor, 1999). Arslan and Kaya (2001) used genetic algorithm to optimize the shape of the membership functions. The base lengths of the input and output fuzzy variables are adjusted to find the optimal membership functions. Garibaldi and Ifeachor (1999) proposed a fuzzy expert system for umbilical cord acid-base interpretation of newborn infants. Analysis of acid-base balance in the blood of umbilical cord gives essential information on any lack of oxygen during childbirth. Opinions of several expert clinicians are sought to rank different complex cases and these rankings are used to train the fuzzy expert system. For optimization purpose, a hybrid of simulated annealing and simplex method are used.

From the review of literature on membership function generation, it is evident that membership grade/membership function generation has received significant research attention over the years. However, there are limited attempts on developing a strategy that combines the best of an expert's knowledge and available data for a better solution. The experience and knowledge of an expert is valuable for initial estimates of a fuzzy parameter, although expert's knowledge may not be fully accurate. Therefore, a fine tuning strategy may be applied to the initial membership grades for finding the optimal membership grades.

In view of it, the objective of the present work is to develop a strategy for fine tuning the initial membership grades by striking a balance between an expert's opinion and accuracy. A methodology is proposed for fine tuning the initial estimates given by an expert to enhance the performance of a fuzzy set based inference system. The proposed methodology is applied in the estimation of burr height in drilling holes. It is used to fine tune the initial membership grades given by the expert and accurately predict the burr height in drilling. Burrs are produced during drilling on both entry and exit surfaces of the workpiece due to plastic deformation of the workpiece material. Formation of burrs during drilling is a critical problem which affects surface quality, dimensional accuracy and safety of handling the product. Burr removal involves extra cost. Therefore, significant amount of research has been devoted towards prediction and control of burr formation in drilling.

Aurich et al. (2009) presented an overview on burr formation in machining operations. A number of case studies on burr formation in turning, milling, drilling and grinding and its control are presented. It is evident from the literature that various parameters affecting burr formation in drilling are material properties, process parameters and drill geometry. Drilling burrs can have different shapes and sizes depending on these parameters. Burr size is highly affected by the ductility of the workpiece material. A number of materials with varying ductility are used for experimental studies on burr formation (Stein and Dornfeld, 1997; Min et al., 2001; Pena et al., 2005; Lauderbaugh, 2009; Ko and Lee, 2001). It is observed that burr height increases with increasing ductility. Effect of process parameters (feed rate and cutting speed) in burr formation in drilling is the most widely studied. Feed rate is found to be a significant factor for burr formation in these studies. Geometry of a drill affects burr shape and size (Min et al., 2001; Lauderbaugh, 2009; Ko and Lee, 2001; Ko et al., 2003). A methodology is proposed to minimize burr size in drilling by using step drills instead of conventional drills (Ko et al., 2003). Effect of cutting speed is not very prominent for burr formation compared to feed rate and ductility of the work material (Min et al., 2001; Lauderbaugh, 2009). In the present work, burr height is considered as a function of ductility, feed rate and drill geometry. Effect of cutting speed on burr formation is not considered in this study.

## 2. Relevance of Membership Grades in Fuzzy Set Theory

To assign suitable values of membership grades to a fuzzy variable and constructing the membership function is one of the most challenging tasks of fuzzy set theory. The membership grade is defined as the degree of being a member of a fuzzy set. Membership grades are subjective, but not arbitrary. In a fuzzy set, the members are allowed to have any positive membership grade between 0 and 1. A membership grade 1 indicates full membership and 0 indicates full non-membership in the set. Any other membership grade between 0 and 1 indicates partial membership of the element in the set. Some skill is needed to form a fuzzy set that properly represents the linguistic name assigned to the fuzzy set. Design of fuzzy membership functions greatly affects a fuzzy set based inference system. Normally an expert’s opinion is sought to construct the membership function for a fuzzy variable. The geometrical shape of the membership function characterizes the uncertainty in the corresponding fuzzy variable. For ease of computation, linear membership functions such as triangular and trapezoidal functions are preferred. However, in order to mimic real life problem, non-linear membership functions may be used.

There are many situations where the membership grades of two or more fuzzy variables are combined to obtain an overall membership grade. For example, consider that a certain job requires sufficient amount of intellectual ability as well as physical fitness. Now, if a candidate has a membership grade of $\mu_{in}$ in the set of 'intellectual' and a membership grade of $\mu_{ph}$ in the set of 'physical fitness', then his/her overall membership grade $\mu_c$ in the set of 'suitable candidates' can be employed using some fuzzy set theoretic operation, such as

$$\mu_c = \min(\mu_{in}, \mu_{ph}) \tag{1}$$

In general, the overall computed/predicted membership grade $\mu_c$ of a fuzzy output variable for $n$ fuzzy input variables can be expressed as

$$\mu_c = f(\mu_1, \mu_2, \ldots\ldots, \mu_n), \tag{2}$$

where $\mu_i$ ($i$=1 to $n$) denotes the membership grade corresponding to $i^{th}$ fuzzy set and $f$ is the appropriate fuzzy set theoretic operation. The success of a fuzzy set based method depends on the accurate assignment of membership grades as well as use of an appropriate fuzzy set theoretic operation. The errors in the estimation of these quantities may reinforce or nullify one another. Hence, it may not be appropriate to apply a fuzzy set based method without the involvement of an expert. However, the estimates of experts may be fine tuned following a systematic mathematical procedure. In this work, it is assumed that the confidence level in the estimation of $\mu_c$ is the highest, followed by the confidence in the appropriateness of $f$. There may be significant uncertainty in the estimation of $\mu_i$ ($i$=1 to $n$) and expert may specify it as a range, rather than a fixed real number. The task is to fine tune the values of $\mu_i$ for satisfying Eq. (2). In doing so, there should not be significant deviation from the opinion of the expert.

## 3. THE METHODOLOGY FOR FINE TUNING MEMBERSHIP GRADES

The membership grades assigned by the expert can be slightly modified based on the observed data. The difference between the computed/predicted and observed overall membership grades can be minimized in the least square sense. The overall methodology comprises the following steps:

a) Data is generated from experiments/ polling/interviews with experts for the fuzzy output variable for which the overall observed membership grade $\mu_o$ is to be obtained.
b) Overall observed membership grade $\mu_o$ is constructed based on the data.
c) Membership grades $\mu_i$ ($i$=1 to $n$) for the fuzzy input variables, their variable bounds, and the appropriate fuzzy set theoretic operator $f$ is selected based on expert's knowledge.
d) Operator $f$ is applied to $\mu_i$ ($i$=1 to $n$) to obtain the value of overall computed membership grade from Eq. (2) which is denoted by $\mu_c$.
e) Objective is to minimize the difference between $\mu_c$ and $\mu_o$ so that observed and computed values of overall membership grades are close to each other giving a suitable solution for the membership grades $\mu_i$ ($i$=1 to $n$) of the fuzzy input variables. The optimization problem is given by Eq. (3) subject to constraints and variable

bounds. In Eq. (3), $k$ is the number of independent observation. The design variables are $\mu_i$ ($i$=1 to $n$), *i.e*. the membership grades of individual attributes.

$$\text{Minimize error } E = \sum_{i=1}^{k} (\mu_c - \mu_o)^2 . \tag{3}$$

f) For fine tuning the membership grades of fuzzy input variables, the following two criteria are considered: (a) accuracy of the solution and (b) deviation of expert's opinion. The accuracy of the solution is expressed in the linguistic form and evaluated as explained below.

The initial estimates of membership grades for the fuzzy input variables and their variable bounds are decided by an expert. The overall membership grade $\mu_c$ is calculated and compared with the overall observed membership grade $\mu_o$. The root mean square (RMS) error value is calculated as per the following equation:

$$\text{RMS error} = \sqrt{\frac{\sum_{i=1}^{k} (\mu_c - \mu_o)^2}{k}} \tag{4}$$

An accurate solution will have a low value of the RMS error. Table 1 shows the RMS errors and their equivalent numerical values. A solution for $\mu_i$ ($i$=1 to $n$) is assigned a numerical value for the level of accuracy attained. A solution with very poor or poor level of accuracy is not acceptable.

**Table 1. The quality of solution based on the accuracy**

| RMS error | Solution quality | Equivalent numerical value |
|---|---|---|
| < 0.08 | Excellent | 10 |
| 0.08–0.1 | Very good | 9 |
| 0.1–0.12 | Good | 8 |
| 0.12–0.15 | Satisfactory | 7 |
| 0.15–0.17 | Poor | 4 |
| >0.17 | Very poor | 2 |

g) If accuracy of the solution is not excellent, the variable bounds of the $\mu_i$ ($i$=1 to $n$) given by the expert are relaxed slightly and a new solution is obtained. For the new solution, each $\mu_i$ is compared with the variable bound provided by the expert and its deviation from the given bound is calculated. For a $\mu_i$ if there is no deviation of the variable bound provided by the expert, it is considered the best. Table 2 shows the numerical values assigned to a $\mu_i$ based on the deviation of expert's opinion.

h) The new solution is also evaluated for accuracy as in Step (f). For an acceptable solution, the minimum level for accuracy as well as deviation of expert's opinion should be satisfactory.

i) Steps (g)) and (h) are repeated and the set of acceptable solutions are obtained. Table 3 shows the numerical values for deviation of expert's opinion and accuracy for each acceptable solution. In Table 3, $e_{ij}$ ($i$=1 to $n$, $j$=1 to $m$) is the numerical value assigned to each $\mu_i$ for deviation of expert's opinion and $E_t$ ($t$=1 to $m$) is the overall quality value calculated for a solution based on deviation of expert's opinion. $A_t$ ($t$=1 to $m$) is the numerical value assigned for the level of accuracy attained by each solution.

**Table 2. The level of deviations of expert's opinion**

| Change in variable bound of a $\mu_i$ given by expert | Level of deviation | Equivalent numerical value |
|---|---|---|
| No change | Excellent | 10 |
| 0.02 | Very good | 9 |
| 0.05 | Good | 8 |
| 0.10 | Satisfactory | 7 |
| 0.15 | Poor | 4 |
| 0.20 | Very poor | 2 |

**Table 3. Evaluation based on accuracy and deviation of expert's opinion**

| Acceptable solutions | Numerical value assigned for deviation of expert's opinion | Overall quality value for a solution ($E_t=\sum e_{ij}/n$) | Numerical value for accuracy ($A_t$) |
|---|---|---|---|
| | $\mu_1, \mu_2, .., \mu_n$ | | |
| 1 | $e_{11}, e_{12},.., e_{1n}$ | $E_1$ | $A_1$ |
| 2 | $e_{21}, e_{22},.. , e_{2n}$ | $E_2$ | $A_2$ |
| . | . | . | . |
| . | . | . | . |
| $m$ | $e_{m1}, e_{m2},.., e_{mn}$ | $E_m$ | $A_m$ |

From the set of acceptable solutions, the solution that satisfies both the criteria with highest possible solution quality is selected as the optimal solution. In some cases, there may be more than one optimal solution leading to a Pareto optimal solution. In a set of Pareto optimal solutions, no solution dominates another solution. In other words, there is no solution in the set which is better (worse) than any other solution from the viewpoint of all the objectives (Dixit and Dixit, 2008).

j) If a satisfactory solution cannot be obtained by the above procedure, there may be a need to modify the operator $f$.

## 4. Burr Height Estimation in Drilling

In this section, the application of the proposed methodology in the estimation of burr height in drilling is described. From the review of literature on burr formation in drilling, it is observed that ductility, feed rate and tool geometry are three significant parameters which affect burr formation in drilling. In the present work, burr height is considered as a function of ductility, feed rate and drill geometry. Effect of cutting speed on burr formation is not considered in this study.

### 4.1. Experimental Work

In the present work, a radial drilling machine (Batliboi Limited, BR618 model) is used for drilling holes in the workpiece. Three different materials of varying ductility, *viz.* aluminium, mild steel and cast iron are used as workpiece material. A two flute high-speed steel drill with 10 mm diameter (118$^o$ point angle and 30$^o$ helix angle) has been used for drilling holes at different feed rates. For each drilling operation, three replicate experiments were performed in the range of feed rate 104–288 mm/min. Spindle speed and cutting velocity are 800 rpm and 25 m/min respectively. The burr height is measured with an Optical Microscope (Axiotech$^{vario}$ 100 HD, make: Carl Zeiss) of magnification range 5X to 200X and supported with KS-300 software. Tables 4–6 show the maximum burr heights for aluminium, mild steel and cast iron work-pieces for four different feed rates respectively. It is observed that for the replicate experiments, the burr height is varying to some extent. This is due to the inherent statistical variation in the machining process.

**Table 4. Burr heights in drilling aluminium**

| Feed rate | Maximum burr height (mm) | | |
|---|---|---|---|
| mm/min | Replicate 1 | Replicate 2 | Replicate 3 |
| 104 | 0.20 | 0.18 | 0.16 |
| 144 | 0.24 | 0.24 | 0.23 |
| 200 | 0.36 | 0.34 | 0.33 |
| 288 | 0.40 | 0.38 | 0.37 |

**Table 5. Burr heights in drilling mild steel**

| Feed rate | Maximum burr height (mm) | | |
|---|---|---|---|
| mm/min | Replicate 1 | Replicate 2 | Replicate 3 |
| 104 | 0.12 | 0.16 | 0.12 |
| 144 | 0.21 | 0.20 | 0.21 |
| 200 | 0.33 | 0.29 | 0.32 |
| 288 | 0.37 | 0.32 | 0.35 |

**Table 6. Burr heights in drilling cast iron**

| Feed rate | Maximum burr height (mm) | | |
|---|---|---|---|
| mm/min | Replicate 1 | Replicate 2 | Replicate 3 |
| 104 | 0.05 | 0.09 | 0.09 |
| 144 | 0.10 | 0.10 | 0.11 |
| 200 | 0.12 | 0.09 | 0.09 |
| 288 | 0.13 | 0.14 | 0.12 |

### 4.2 Application of the Proposed Methodology

In this section, the proposed methodology is applied in burr height estimation in drilling. To represent different membership grades for burr heights (data obtained from the experiments), the standard *S*-function is selected. Eq. (5) represents the standard *S*-function (Zadeh, 1976).

$$S(\mu_o; a, b, c) = \begin{cases} 0 & \text{if } \mu_o \le a \\ 2\left(\dfrac{\mu_o - a}{c-a}\right)^2 & \text{if } a < \mu_o \le b \\ 1 - 2\left(\dfrac{\mu_o - c}{c-a}\right)^2 & \text{if } b < \mu_o \le c \\ 1 & \text{if } \mu_o > c \end{cases} \tag{5}$$

where $b=(a + c)/2$. In Eq. (5), $\mu_o$ represents overall membership grade for observed burr heights and *a*, *b*, and *c* are the values of burr heights at membership grades 0, 0.5 and 1 respectively. From the experimental study, the maximum and minimum values of burr heights are found as 0.40 mm (aluminium workpiece at 288 mm/min feed rate) and 0.05 mm (cast iron workpiece at 104 mm/min feed rate). Based on these values, parameter *a* (value of burr height at membership grade 0) is taken as 0.02 mm below which burr height is considered negligible. Parameter *c* (the value of burr height at membership grade 1) is taken as 0.5 mm and parameter *b* (the value of burr height at membership grade 0.5) is obtained as 0.26 mm.

Figure 1 shows the overall membership grades $\mu_o$ for the observed burr heights. The value of $\mu_o$ for the maximum burr height 0.40 mm is 0.913 and that for minimum burr height 0.05 mm is 0.008.

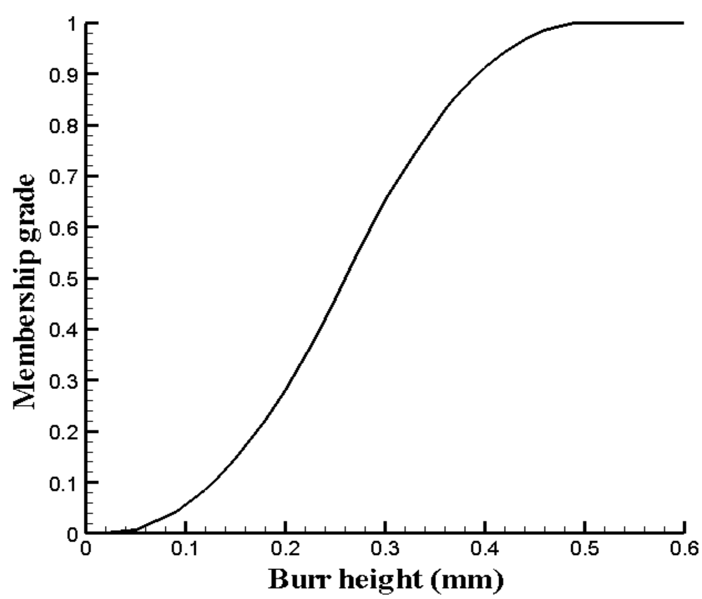


Figure 1. Membership function for observed burr heights

For aluminium, the maximum variation of burr height for replicate experiments is 0.04 mm for which the variation in the value of $\mu_o$ is 0.11. Thus, there may be an error of the order of 0.11 in the estimation of overall membership grade for aluminium. For mild steel and cast iron, the maximum variations of burr height for replicate experiments are 0.05 mm and 0.04 mm and errors in the value of $\mu_o$ for mild steel and cast iron may be 0.13 and 0.04 respectively.

From the knowledge acquired from the literature, it is observed that ductility of the workpiece material, feed rate and tool geometry are the three significant parameters that affect burr formation in drilling. The input parameters may have varying effect on burr height. The ductility of the work material plays a dominant role compared to the other two parameters. To take into account the varying effect of the input parameters, the following relation is adopted. If $\mu_{duc}, \mu_{tool}$ and $\mu_{feedrate}$ are the individual membership grades (ranging from 0 to 1) assigned to ductility of the workpiece material, tool geometry and feed rate respectively, then it is proposed to calculate the overall membership grade $\mu_c$ for burr height as

$$\mu_c = \mu_{duc}\left\{\left(\frac{\mu_{tool}+\mu_{feedrate}}{2}\right)\wedge 1\right\}, \tag{6}$$

which asserts the greater effect of ductility and combined additive effect of tool geometry and feed rate on burr height in drilling. Note that $(a \wedge b)$ indicates minimum of *a* and *b*.

The initial values of the $\mu_i$ ($\mu_{duc}$, $\mu_{feedrate}$ and $\mu_{tool}$) and their variable bounds are provided by the expert. For three different materials, aluminium (Al), mild steel (MS), and cast iron (CI) of varying ductility, and three different feed rates, the values of $\mu_i$ are given in Table 7. In the experimental work, a conventional drill with 118$^o$ point angle is used for drilling operation. For conventional drills, burr height is found more (Ko et al., 2003). Therefore tool geometry ($\mu_{tool}$) is assigned the membership grade 0.9 and its variable bound is 0.8–1. The overall membership grade $\mu_c$ for burr height is calculated from Eq. (6) for all the combinations of workpiece material, feed rate and tool geometry. The objective function given by Eq. (3) is minimized using optimization technique FMINCON in MATLAB (Version 7). FMINCON attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate. The design variables are the membership grades of individual attributes, *i.e.* $\mu_{duc1}$, $\mu_{duc2}$, $\mu_{duc3}$, $\mu_{feedrate1}$, $\mu_{feedrate2}$, $\mu_{feedrate3}$ and $\mu_{tool}$. Following the methodology described in Section 3, each solution is evaluated for accuracy and deviation of expert's opinion. Table 8 shows the acceptable solutions satisfying the criteria that the minimum level for accuracy as well as deviation of expert's opinion should be satisfactory. Between Solutions 1 and 2, Solution 1 is better. Among the Solutions 3–7, Solution 3 is the best as it dominates the other solutions. However, between Solutions 1 and 3, no solution dominates the other. Both the solutions form a set of Pareto optimal solution from the viewpoint of satisfying the criteria for accuracy and deviation of expert's opinion. A higher level of decision is required to choose between these two solutions. Table 9 shows the values of design variables for Solution 1 and 3.

**Table 7. Input parameter membership grades and variable bounds given by the expert**

| $\mu_{duc}$ | | | Variable bound | Feed rate | $\mu_{feedrate}$ | | Variable bound |
|---|---|---|---|---|---|---|---|
| Al | $\mu_{duc1}$ | 0.9 | 0.75–0.95 | 288 | $\mu_{feedrate1}$ | 0.8 | 0.75–0.90 |
| MS | $\mu_{duc2}$ | 0.7 | 0.60–0.75 | 200 | $\mu_{feedrate2}$ | 0.5 | 0.45–0.60 |
| CI | $\mu_{duc3}$ | 0.2 | 0.15–0.25 | 104 | $\mu_{feedrate3}$ | 0.3 | 0.15–0.30 |

**Table 8. Acceptable solutions based on accuracy and deviation of expert's opinion**

| Solution | Overall value for $\mu_i$s for deviation of expert's opinion ($E_t$) | Numerical value assigned for accuracy ($A_t$) |
|---|---|---|
| **1** | **9.43** | **7** |
| 2 | 8.57 | 7 |
| **3** | **8** | **8** |
| 4 | 7.86 | 8 |
| 5 | 7.71 | 8 |
| 6 | 7.43 | 8 |
| 7 | 7.14 | 8 |

**Table 9. The optimal solutions for the membership grades of input parameters**

| Input parameter $\mu_i$ | Solution-1 | Solution-3 |
|---|---|---|
| $\mu_{duc1}$ | 0.95 | 0.99 |
| $\mu_{duc2}$ | 0.80 | 0.80 |
| $\mu_{duc3}$ | 0.10 | 0.10 |
| $\mu_{feedrate1}$ | 0.90 | 0.95 |
| $\mu_{feedrate2}$ | 0.60 | 0.70 |
| $\mu_{feedrate3}$ | 0.15 | 0.05 |
| $\mu_{tool}$ | 0.86 | 0.81 |

For validation of the proposed method, drilling experiments were performed at an intermediate feed rate of 144 mm/min. The maximum burr heights of three replicate experiments for aluminium at feed rate 144 mm/min were found as 0.24 mm, 0.24 mm and 0.23 mm as shown in Table 10. Corresponding overall membership grade $\mu_o$ of these observed burr heights are 0.42, 0.42 and 0.38. The predicted membership grade $\mu_c$ is 0.54 with the expert's initial estimate of $\mu_i$ ($i$=1 to $n$) which gives an error of 22.22%, 22.22% and 29.62% compared to the observed $\mu_o$ values of the three replicate experiments respectively. In order to give more importance to solution accuracy, the fine tuned values of $\mu_i$ are taken from Solution-3 of Table 9. The value of $\mu_c$ is 0.43 with the fine tuned values giving an error of 2.33%, 2.33% and 11.63% compared to the observed $\mu_o$ values of the replicate experiments. Thus, there is a better matching of $\mu_c$ and $\mu_o$ values with fine tuned values of $\mu_i$ than with initial expert's values of $\mu_i$. For mild steel, the burr heights of three replicate experiments were found as 0.21 mm, 0.20 mm and 0.21 mm with corresponding values of $\mu_o$ as 0.31, 0.28 and 0.31 (Table 11). The value of $\mu_c$ (0.42) with the initial expert's values of $\mu_i$ gives an error of 26.19%, 33.33% and 26.19% compared to the observed $\mu_o$ values whereas the value of $\mu_c$ (0.34) with fine tuned values of $\mu_i$ gives an error of 8.82%, 17.65% and 8.82%. For cast iron also, similar results were observed. Thus it is observed that in all the three cases, the fine tuned values of $\mu_i$ ($i$=1 to $n$) give better matching of $\mu_c$ and $\mu_o$ than the initial expert's values of $\mu_i$. Fine tuning of the initial expert's estimates has enhanced the performance of the burr height prediction methodology.

**Table 10. Comparison of $\mu_c$ with $\mu_o$ (predicted and observed burr heights) in drilling in aluminium at feed rate 144 mm/min**

| Replicates | Maximum burr height (mm) | Observed membership grade $\mu_o$ for maximum burr height | Predicted membership grade $\mu_c$ with expert's values of $\mu_i$ | % error of $\mu_c$ (with expert's values) compared to $\mu_o$ | Predicted membership grade $\mu_c$ with fine tuned values of $\mu_i$ | % error of $\mu_c$ (with fine tuned values) compared to $\mu_o$ |
|---|---|---|---|---|---|---|
| Replicate1 | 0.24 | 0.42 | 0.54 | 22.22 | 0.43 | 2.33 |
| Replicate2 | 0.24 | 0.42 | 0.54 | 22.22 | 0.43 | 2.33 |
| Replicate3 | 0.23 | 0.38 | 0.54 | 29.62 | 0.43 | 11.63 |

**Table 11. Comparison of $\mu_c$ with $\mu_o$ (predicted and observed burr heights) in drilling in mild steel at feed rate 144 mm/min**

| Replicates | Maximum burr height (mm) | Observed membership grade $\mu_o$ for maximum burr height | Predicted membership grade $\mu_c$ with expert's values of $\mu_i$ | % error of $\mu_c$ (with expert's values) compared to $\mu_o$ | Predicted membership grade $\mu_c$ with fine tuned values of $\mu_i$ | % error of $\mu_c$ (with fine tuned values) compared to $\mu_o$ |
|---|---|---|---|---|---|---|
| Replicate1 | 0.21 | 0.31 | 0.42 | 26.19 | 0.34 | 8.82 |
| Replicate2 | 0.20 | 0.28 | 0.42 | 33.33 | 0.34 | 17.65 |
| Replicate3 | 0.21 | 0.31 | 0.42 | 26.19 | 0.34 | 8.82 |

## CONCLUSION

Assigning suitable values of membership grades to a fuzzy variable and constructing the membership function is a challenging task in fuzzy set theory. Design of fuzzy membership functions greatly affects a fuzzy set based inference system. Normally an expert's opinion is sought to construct the membership function for a fuzzy variable. The experience and knowledge of an expert is valuable for initial estimates of a fuzzy parameter. However, there is a need to optimize these estimates to enhance performance. In this work, a methodology is developed for fine tuning the initial membership grades assigned by an expert for fuzzy set based inference system.

The proposed methodology combines the best of an expert's knowledge and available data to find the optimal values for membership grades. The proposed methodology is applied in the estimation of burr height in drilling holes. It is observed that the fine tuned values of membership grades give better matching of predicted and observed membership grades for burr height than the initial expert's values of membership grades. Fine tuning of the initial expert's estimates has enhanced the performance of the burr height prediction methodology. The methodology is suitable where limited information is available initially and information value keeps on increasing.

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