Assignment - I

First-order PDE. [Ref. I. N. Sneddon]

1. Construct a PDE by eliminating arbitary constants a & b from

   a) $(x-a)^2+(y-b)^2+z^2=r^2$

   b) $z-ax-by-ab=0$

   c) $x^2+y^2-(z-a)^2=b^2$

2. Construct a PDE by eliminating the arbitary function f rom

   a) $u = x+y+f(xy)$ $[px-qy=x-y]$

   b) $u = f(y/x)$ $[px+qy=0]$

   c) $u = f(x^2+y^2)$ $[py-xq=0]$

3. If $F(\phi,\psi)=0$ is an arbitary relation between the two known functions $\phi=\phi(x,y,z)$ and $\psi=\psi(x,y,z)$, then show that $z=z(x,y)$ satisfy the PDE
$$p\,\frac{\partial(\phi,\psi)}{\partial(y,u)}+q\,\frac{\partial(\phi,\psi)}{\partial(v,x)}=\frac{\partial(\phi,\psi)}{\partial(x,y)}.$$

4. Find the general solution of

   a) $y^2zp+z^2xq=-xy^2$

   b) $y^2zp-xyq=x(z-2y)$

   c) $(y+z)p+yq=(x-y)$

   d) $(y+2zx)p-(x+2zy)q=\frac{1}{2}(x^2-y^2)$

   e) $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$.

5. Find the particular integral of
$$2y(z-3)p + (2x-z)q = y(2x-3),$$
which passes through the circle
$$z=0,\quad x^2+y^2=2x.$$

6. $$u_t + 4u_x = u^2$$
$$u(x,0) = \frac{1}{1+x^2}$$

7. $$-4x\,u_z + 9t\,u_x = xt/4$$

8. $$u\,u_t + x u_x = t,\quad u(1,t) = 2t.$$

9. $$(x-y)y^2p + (y-x)x^2q = (x^2+y^2)z$$
passing through the curve, $xz = a^3,\ y=0$.

10. $u_x + 2x u_y = u^2$, find general solution

11. $(u^2+y^2)u_x - xy u_y + xu = 0$, " "

12. $u(u^2+xy)(x u_x - y u_y) = x^4$, " ".

13. Find the surface which is orthogonal to the one-parameter family of surfaces $z = cxy(x^2+y^2)$ and passes through the curve: $x^2-y^2=a^2, z=0$.

14. Find the complete integral
a) $p(1+q^3) = q(z-a)$
b) $xp + yq = z - a\sqrt{x^2+y^2+z^2}$
c) $pz - qz = z^2 + (x+y)^2$
d) $x^2p^2 + y^2q^2 = z^2$
e) $(x+y)^2(p+q)^2 + (x-y)(p-q)^2 = 1$
f) $(x^2+y^2)(p^2+q^2) = 1$, g) $z^2(p^2z^2+q^2) = 1$,
h) $z^2(p^2+q^2) = x^2+y^2$, i) $z^2(p^2+q^2+1) = c^2$

15. Show that the following PDEs are compatible
$xp - yq = x$, $x^2p + q = xz$, and find the solution.

[Ref. Elements of PDE by I. N. Sneddon]