MA20103 - Partial differential equations

Problem Sheet [*]

October 20, 2017

1. Find the general solution of the following equations:

   (a) $(3D^2 - 2DD' - 5D'^2)z = 3x + y + e^{x-y}$,

   (b) $(2D^2 - 5DD'5D'^2)z = 0$,

   (c) $r + s - 2t = e^{x+y}$,

   (d) $r - s + 2q - z = x^2y^2$,

   (e) $r + s - 2t - p - 2q = 0$.

2. Find the particular integral of the following partial differential equations(Try with different methods):

   (a) $(3D^2 - D')z = e^x sin(x + y)$,

   (b) $(D^2 - D')z = 17e^{x+y} sin(x - 2y)$,

   (c) $(D^2 + D')z = 6xy + 25e^{3x+4y}$,

   (d) $(D^2 + D'^2 - D)z = 37e^{5y} cos(3x + 4y)$,

3. Classify and reduce the following equations to canonical forms and hence solve(if possible)

   (a) $y^2 Z_{xx} - x^2 z_{yy} = 0$,

   (b) $x^2 z_{xx} + 2xy z_{xy} + y^2 z_{yy} = 0$,

   (c) $z_{xx} + x^2 z_{yy} = 0$,

   (d) $e^{2x} z_{xx} - 2e^{x+y} z_{xy} + e^{2y} z_{yy} + e^{2x} z_x + e^{2y} z_y = 0$,

   (e) $e^{2x} z_{xx} - 5e^{x+y} z_{xy} + 4e^{2y} z_{yy} + e^{2x} z_x + 4e^{2y} z_y = 0$,

   (f) $9z_{xx} - 12z_{xy} + 4z_{yy} + 12z_x - 8z_y + 4z = 0$,

   (g) $3z_{xx} - 7z_{xy} + 2z_{yy} + 3z_x - z_y = 0$,

   (h) $2z_{xx} + 6z_{xy} + 9z_{yy} + 2z_x + 3z_y - 2z = 0$,

   (i) $xz_{xx} + 2x^2 z_{xy} = z_x - 1$,

(j) $x^2 z_{xx} + 2x z_{xy} + z_{yy} = z_y$.

4. Solve the following partial differential equations:

   (a) $z_{xx} - 4z_{xy} - 6z_{yy} = 0$, $z(0, y) = sin3y$, $z_x(0, y) = sin2y$,

   (b) $z_{xx} - 4z_{xy} = 0$, $z(0, y) = y^2$, $z_x(0, y) = 1 - 3y$.