Partial Differential Equations (MA20103)

Assignment -2

First order PDE

1. Form the partial differential equation by eliminating arbitrary constants of a and b from the equation $2z=(ax+y)^2+b$.

2. Form the partial differential equation by eliminating the arbitrary constants h and k from the equation $(x-h)^2+(y-k)^2+z^2=\lambda^2$.

3. Form the partial differential equation by eliminating the arbitrary function f from the relation $z=xy+f\left(x^2+y^2\right)$.

4. Eliminate the arbitrary function F from the relation $F\left(x^2+y^2+z^2, z^2-2xy\right)=0$ and form the corresponding PDE.

5. Classify the following first order PDE.
(i) $p+q-z=xy$
(ii) $\left(x^2-yz\right)p+\left(y^2-zx\right)q=z^2-xy$
(iii) $x^2p^2+y^2q^2-z^2=0$
(iv) $y^2(yp+xq)=x^2z^2$
(v) $pq=z$

6. Solve the following 1st order PDE by Lagrange's method

(i) $zp=-x$

(ii) $\left(x^2+2y^2\right)p-xyq=xz$

(iii) $z\left(z^2+xy\right)(px-qy)=x^4$

(iv) $xzp+yzq=xy$

(v) $xp-yq=xy$

(vi) $x\left(y^2-z^2\right)q-y\left(z^2+x^2\right)q=z\left(x^2+y^2\right)$

(vii) $x\left(y^2+z\right)p-y\left(x^2+z\right)q=z\left(x^2-y^2\right)$

(viii) $\left(y-z\right)p+\left(z-x\right)q=x-y$

(ix) $\left(y+z\right)p+\left(z+x\right)q=x+y$

(x) $p+q=x+y+z$

7. Find the equation of the integral surface of the differential equation $2y\left(z-3\right)p+\left(2x-z\right)q=y\left(2x-3\right)$
which passes through the circle $z=0,\ x^2+y^2=2x$.

8. Find the surface which is orthogonal to the one parameter system $z=cxy\left(x^2+y^2\right)$ and which passes through the hyperbola $x^2-y^2=a^2,\ z=0$.

9. If $u_1=\frac{\partial u}{\partial x}, u_2=\frac{\partial u}{\partial y}, u_3=\frac{\partial u}{\partial z}$ show that the equation $f\left(x,y,z,u_1,u_2,u_3\right)=0$ and $g\left(x,y,z,u_1,u_2,u_3\right)=0$ are compatible if $\frac{\partial\left(f,g\right)}{\partial\left(x,u_1\right)}+\frac{\partial\left(f,g\right)}{\partial\left(y,u_2\right)}+\frac{\partial\left(f,g\right)}{\partial\left(z,u_3\right)}=0$.

10. Show that the equations $xp-yq=x$ and $x^2p+q=xz$ are compatible and find their solution.

11. Show that the equations $x=yq$ and $z\left(xp+yq\right)=2xy$ are compatible and solve them.

12. Use Charpit's method to solve the following PDE :

(i) $\left(p^2+q^2\right)y=qz$

(ii) $z^2=pqxy$

(iii) $p=\left(z+qy\right)^2$

(iv) $p+q=pq$

(v) $z=p^2-q^2$

(vi) $p^2q\left(x^2+y^2\right)y=p^2+q$

(vii) $z=ax+by+\frac{a^4+b^4}{ab}$

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