Problem set 4

Spring 2018

MATHEMATICS-II (MA10002)(Numerical Analysis)

1. Solve the following equations by (i) Gauss-Jacobi method (ii) Gauss-Seidel method, correct up to three decimal places with initial guess a) (0,0,0); b) (0,0,0,0) and c) (0,0,0,0) respectively

   (a)

$$\begin{aligned} 7x + 2y - z &= 17.20 \\ -x + 9y + 2z &= 18.90 \\ x + 5y - 11z &= 28.05 \end{aligned}$$

   (b)

$$\begin{aligned} 10x_1 - 2x_2 - x_3 - x_4 &= 3 \\ -2x_1 + 10x_2 - x_3 - x_4 &= 15 \\ -x_1 - x_2 + 10x_3 - 2x_4 &= 27 \\ -x_1 - x_2 - 2x_3 + 10x_4 &= -9 \end{aligned}$$

   (c)

$$\begin{aligned} 13x_1 + 5x_2 - 3x_3 + x_4 &= 18 \\ 2x_1 + 12x_2 + x_3 - 4x_4 &= 13 \\ 3x_1 - 4x_2 + 10x_3 + x_4 &= 29 \\ 2x_1 + x_2 - 3x_3 + 9x_4 &= 31 \end{aligned}$$

2. Find a real root, lying between 2 and 3, of the equation $x^3 - x - 11 = 0$ by using bisection method correct up to three decimal places.

3. Solve the equations

   (a) $2x - 3\sin x - 5 = 0$

   (b) $x \log_{10} x = 1.2$

   by bisection method for the root lying between 2 and 3 correct up to three decimal places.

4. For an equation $f(x) = 0$, discuss the convergence criteria of fixed point iteration method, of the presentation $x = \phi(x)$ of the given equation $f(x) = 0$.

5. Find the root of the equation: $5x^3 - 20x + 3 = 0$ by using fixed point iteration correct up to three decimal places.

6. Find the root of the equation: $\sin x = 10(x - 1)$ by using fixed point iteration correct up to three decimal places.

7. Find the root of the equation: $x^2 + \ln x - 2 = 0$ which lies between 1 and 2, by using i) fixed point iteration method, ii) Newton-Raphson method, correct up to three decimal places. Then compare the number of iterations used in both the cases.

8. Find the root of the equation by Newton-Raphson method: $x \sin x + \cos x = 0$, near π.

9. Find a positive root lying between 0.5 to 0.8, of $10^x + x - 4 = 0$ by Newton-Raphson method, correct up to six decimal places.

10. Find the iterative formula for finding $\sqrt[3]{N}$, where N is a positive real number, using Newton-Raphson method. Hence evaluate $\sqrt[3]{10}$ correct upto four places of decimal.

11. From the equation $x^5 - a = 0$, deduce the Newton-Raphson iterative procedure:

$$x_{n+1} = \tfrac{1}{5}[4x_n + \tfrac{a}{x_n^4}] \text{ for } \sqrt[5]{a}.$$

Use this formula to evaluate $\sqrt[5]{3}$, correct upto four decimal places.

12. Find a double root (i.e. m=2) for the equation $x^3 - x^2 - x + 1 = 0$, using:

i) $x_{n+1} = x_n - m\frac{f(x_n)}{f'(x_n)}$

ii) Newton-Raphson method, correct upto three decimal places.

Then compare the number of iteration used in both the cases.