Tutorial Sheet - 5

SPRING 2017

MATHEMATICS-II (MA10002)(Numerical Analysis) *January 13, 2017*

1. Solve the following system of equations by Gauss-elimination method.

   (a)

$$\begin{aligned} x_1 + 4x_2 - x_3 &= -5 \\ x_1 + x_2 - 6x_3 &= -12 \\ 3x_1 - x_2 - x_3 &= 4 \end{aligned}$$

   (b)

$$\begin{aligned} 6.32x_1 - 1.73x_2 - 0.65x_3 + 1.06x_4 &= 2.95 \\ 1.13x_1 - 0.89x_2 + 0.61x_3 + 5.63x_4 &= 4.27 \\ 0.74x_1 + 1.01x_2 + 5.28x_3 - 1.88x_4 &= 1.97 \\ 0.89x_1 + 4.32x_2 - 0.47x_3 + 0.95x_4 &= 3.36 \end{aligned}$$

2. Solve the following equations by (i) Gauss-Jacobi method (ii) Gauss-Seidel method, correct upto four decimal places.

   (a)

$$\begin{aligned} 6.32x_1 - 0.73x_2 - 0.65x_3 + 1.06x_4 &= 2.95 \\ 0.89x_1 + 4.32x_2 - 0.47x_3 + 0.95x_4 &= 3.36 \\ 0.74x_1 + 1.01x_2 + 5.28x_3 - 0.88x_4 &= 1.97 \\ 1.13x_1 - 0.89x_2 + 0.61x_3 + 5.63x_4 &= 4.27 \end{aligned}$$

   (b)

$$\begin{aligned} 4.50x_1 + 0.15x_2 + 0.30x_3 &= 1.57 \\ 0.15x_1 - 10.50x_2 + 0.45x_3 &= -3.86 \\ 0.45x_1 + 0.30x_2 - 15.00x_3 &= 14.28 \end{aligned}$$

(c)

$$\begin{aligned}
2.38x_1 + 1.95x_2 - 8.27x_3 + 1.58x_4 &= 2.16\\
3.21x_1 - 0.86x_2 + 2.42x_3 - 7.20x_4 &= 3.28\\
1.44x_1 + 6.95x_2 - 2.14x_3 + 1.86x_4 &= 1.42\\
9.17x_1 + 3.62x_2 - 1.68x_3 - 2.26x_4 &= 5.21
\end{aligned}$$

3. Find a root of the equation $x^3 - 4x - 9 = 0$, using Bisection method, correct upto 4-decimal places.

4. Solve the equation $x^3 - 9x + 1 = 0$ by Bisection method for the root lying between 2 and 3, correct upto 3-significant figures.

5. Find the positive root of $x^3 + x - 1 = 0$, by fixed point iteration method, correct upto four decimal places.

6. Find the root of $x^2 + lnx - 2 = 0$ which lies between 1 and 2, by fixed point iteration method, correct upto four decimal places.

7. Find a real root of $3x = cosx + 1$, by Newton-Raphson method, with an initial guess of $x_0 = 0.6$.

8. Find a real root of $x^x + x - 4 = 0$, by Newton-Raphson method, correct to six decimal places, with an initial guess of 1.6.

9. Find the double root of the equation $x^3 - x^2 - x + 1 = 0$, by using

   (a) $x_{n+1} = x_n - m\dfrac{f(x_n)}{f'(x_n)}$.

   (b) Newton-Raphson method.

   with an initial guess of $x_0 = 0.9$. Compare the number of iterations.

10. Obtain Newton-Raphson extended formula

    $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} - \dfrac{1}{2}\dfrac{\{f(x_0)\}^2 f''(x_0)}{\{f'(x_0)\}^2}$ for the root of the equation $f(x) = 0$.

11. The equation $x = f(x)$ is solved by the iteration method $x_{k+1} = f(x_k)$ and a solution is wanted with a maximum error not greater than 0.5×10^{-4}. The first and second iterations were computed as : $x_1 = 0.50000$ and $x_2 = 0.52661$. How many iterations must be performed further, if it is known that $|f'(x)| \leqslant 0.53$ for all values of x.

12. Find the interval in which the smallest positive root of the following equation lies: $\tan x + \tanh x = 0$

13. Find the n-th root of a positive real number a. Hence find $\sqrt{18}$.

14. The root of the equation $x = \frac{1}{2} + \sin x$ by using the iteration method $x_{k+1} = \frac{1}{2} + \sin x_k,\ k = 0, 1, 2, 3 \ldots$ with $x_0 = 1$ correct to six decimals is $x = 1.497300$. Determine the number of iteration steps required to reach the root by fixed point iteration method.

15. Find all positive roots to the equation $10 \int_0^x e^{-x^2} dt = 1$ correct upto six decimal places.