MA20103 - Partial differential equations
Assignment 4

October 21, 2016

1. Employing Charpit's method, find the complete integral of the following equations :

   (a) $p^2x + q^2y = z$

   (b) $px^5 - 4q^3x^2 + 6x^2z - 2 = 0$

   (c) $2(z + xp + yq) = yp^2$

   (d) $p + q = pq$

   (e) $p^2q - 2x^2q = x^2y$

   (f) $p^2q^2 + x^2y^2 = x^2q^2(x^2 + y^2)$

2. Find the complete integral of each of the following equations and also any singular solution that may exist :

   (a) $x^2p^2 + y^2q^2 = 1$

   (b) $pq + z = 1$

3. Verify that $z = ax + by + a + b - ab$, where a, b are arbitrary constants, is a complete integral of the p.d.e $z = px + qy + p + q - pq$. Show that the envelope of all planes corresponding to the complete integral, provides a singular solution of the differential equation. Also determine a particular solution by finding the envelope of those planes that pass through the origin.

4. Show that $(x - a)^2 + (y - b)^2 + z^2 = 1$ is a complete integral of $z^2(1 + p^2 + q^2) = 1$. By taking $b = 2a$, show that the envelope of the sub-family is $(y - 2x)^2 + 5z^2 = 5$ which is a particular solution. Also find the singular solution(s) if they exist.

5. Solve the following second-order linear partial differential equations with constant co-efficients:

   (a) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = 0$

   (b) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = e^{x+y}$

(c) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \sin x \cos 2y$

(d) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = xy$

(e) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} - z = x^2 y^2$

(f) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} - z - \frac{\partial z}{\partial x} - 2\frac{\partial z}{\partial y} = 0$

6. Reduce the following equations to canonical form and solve whenever possible:

   (a) $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + 17\frac{\partial^2 z}{\partial y^2} = 0$

   (b) $(n-1)^2 \frac{\partial^2 z}{\partial x^2} - y^{2n} \frac{\partial^2 z}{\partial y^2} = n y^{2n-1} \frac{\partial z}{\partial y}$

   (c) $y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x} \frac{\partial z}{\partial x} + \frac{x^2}{y} \frac{\partial z}{\partial y}$

   (d) $x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} - 2y \frac{\partial z}{\partial y} = 0$

   (e) $y^2 \frac{\partial^2 z}{\partial x^2} + 2x^2 \frac{\partial^2 z}{\partial y^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y \frac{\partial z}{\partial y} = 0$