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Measuring near-saturated hydraulic conductivity of soils by quasi unit-gradient percolation—1. Theory and numerical analysis

Subharthi Sarkar[1], **Kai Germer**[1], **Rajib Maity**[1], and **Wolfgang Durner**[2]*

[1] Department of Civil Engineering, Indian Institute of Technology Kharagpur, Kharagpur–721302, West Bengal, India
[2] Division of Soil Science and Soil Physics, Institute of Geoecology, Technische Universität Braunschweig, Langer Kamp 19c, Braunschweig, Lower Saxony, 38106, Germany

Abstract

The saturated and near-saturated hydraulic conductivity of soils, k_u, is a sensitive indicator of soil structure and a key parameter for solute transport and soil aeration. In this contribution, we present and numerically investigate a double-disk method to determine k_u in the laboratory by steady-state percolation at different suction steps. Tension infiltration of water takes place at the top of a soil column through a porous disk with a smaller diameter than the soil sample. This leaves part of the soil surface open and ensures a proper soil ventilation. Drainage takes place at the base through a porous disk with the full diameter of the soil column at exactly the same tension as applied to the top boundary. Since the infiltration area is less than the percolation area, the water flow diverges and the equality of steady flow rate and hydraulic conductivity, which characterizes the standard unit-gradient experiment, is no longer valid. To develop a general relationship between observed steady flow rate and unsaturated hydraulic conductivity, the experiment was simulated with the Richards-equation solver HYDRUS 2D/3D, for twelve different soil classes. We found for tensions in the range 1 cm < 10 cm, an infiltration disk diameter of 4.5 cm diameter and a sample diameter of 8 cm diameter that the flux rate at any given tension was about 0.7 times the respective hydraulic conductivity, with an error of less than 10%.

Key words: HYDRUS 2D/3D / measurement method / near saturated hydraulic conductivity / tension disk infiltrometer / unit-gradient experiment

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1 Introduction

Soil is a complex three-phase porous medium with hydraulic storage and transmission properties that are highly nonlinearly dependent on the water content. In the framework of the continuum approach, these properties are defined by hydraulic functions, namely the water retention curve and the hydraulic conductivity curve (*e.g.*, *Durner* and *Flühler*, 2005). Parameterized hydraulic functions are required for modelling water and solute transport in the vadose zone. Their proper characterization, both in saturated and unsaturated state, plays an important role in addressing a variety of problems in hydrology, ecology, environmental sciences, soil science, agriculture, and many other disciplines (*Vereecken* et al., 2016).

The soil hydraulic conductivity, k_u, expressed either as function of the volumetric water content, θ, or of the matric potential head, h, is a key factor in unsaturated flow problems, such as the design and performance assessment of irrigation and drainage systems, earthen waste impoundments and many other agricultural, geotechnical, and environmental structures (*Reynolds* et al., 2000). Detailed information on the soil hydraulic conductivity close to saturation is particularly important for simulating transport of water and chemicals in soils. Near saturation, it can change dramatically due to the presence of macropores (*e.g.*, *Germer* and *Braun*, 2015), which is difficult to handle in traditional pore-size distribution models (*Børgesen* et al., 2006).

Several methods are available to measure the hydraulic conductivity of soil in saturated and unsaturated state, which can be broadly classified into two categories: direct measurement methods and indirect estimation methods (*van Genuchten* et al., 1991). Because of the difficulty and different shortcomings of direct measurement procedures, indirect estimation methods are predominant (*Durner* and *Lipsius*, 2005). However, all the estimation procedures need direct measurements as benchmarks for validation of their results (*Dirksen*, 2000). Moreover, for structured soils the prediction of near saturated $k_u(h)$ from soil texture or from the water retention curve is fundamentally unreliable (*Durner*, 1994).

In the field, k_u is often estimated by tension infiltrometry, but the calculation of hydraulic conductivity from an unconstrained three-dimensional transient flow field relies on a variety of assumptions with respect to the homogeneity of the soil below the infiltrometer, initial water content, and the type of

* Correspondence: W. Durner; e-mail: w.durner@tu-bs.de

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hydraulic functions, which might not always be fulfilled. In the lab, direct measurement of unsaturated conductivity in the wet range is rather uncommon. Available methods are the traditional double membrane method (*Henseler* and *Renger*, 1969), the crust method (*Bouma* et al., 1983), which is mostly used in the field, but also applicable in the laboratory, the multi-step outflow method (MSO) (*Eching* et al., 1994; *Hopmans* et al., 2002), and the Multi-Step Flux method (MSF) (*Weller* et al., 2011), where water is applied at the top of a soil column by irrigation and drained at the bottom by suction. Each of these methods has its specific difficulties, problems and pitfalls. The crust method, *e.g*., requires for each flux density the preparation of a specific crust, which is not easy to specify *a priori*. The MSO is an established method for determining simultaneously water retention and conductivity properties from saturation to medium dry conditions, but has a very low sensitivity towards K near saturation (*Iden* and *Durner*, 2007). The problem results from the fact that the amount of water that is available to percolate and thus characterize the conductivity at saturation and close to saturation is very small, and the duration of the transition from full saturation to unsaturated conditions is very short. This means that the sensitivity towards a highly resolved conductivity function near saturation is not very good. This is fundamentally different for steady percolation methods. For that reason, *Durner* and *Iden* (2011) developed the XMSO experiment, where the outflow is preceded by a percolation phase, which partly helps to resolve the situation. Still, the XMSO is not trivial to perform and evaluate. Finally, the MSF method requires the pressure head at the bottom to be adjusted to a value corresponding to the pressure head measured in the soil near the top. This requires iterative re-adjustment of the bottom suction that is done either manually (*Dirksen*, 1999) or by a computer algorithm (*Weller* et al., 2011) and may require appreciable time.

Summarizing, all available methods have in common that their realization is delicate, laborious and difficult, and no commercial apparatus exists for that purpose. Near saturation, the pore structure becomes most fragile and there is a risk that the measurement process itself changes the system (*Ghezzehei* and *Or*, 2003).

Due to these experimental difficulties, it is common practice to estimate $k_u(h)$ by a shape function of the relative conductivity function, $k_r(h)$, which is predicted from the water retention curve that reflects an effective pore-size distribution. The corresponding function is either matched to the saturated conductivity, k_s, yielding $k_u(h) = k_s \times k_r(h)$, or estimated by extrapolation from unsaturated conductivity data that have been measured in a dryer range, *e.g*., with the evaporation method (*Wendroth* et al., 1993). These estimation approaches are problematic due to the extreme dependency of the prediction of the near-saturated hydraulic conductivity on the pore-size distribution and pore structure (*Vogel* and *Cislerova*, 1988; *Durner*, 1994), which causes an enormous variability of k_s in natural structured soils.

The objective of this study is to develop a measurement method for unsaturated hydraulic conductivity at small negative pressure heads (subsequently called "suctions") by steady-state percolation of water that is applied by tension infiltration onto a laboratory column. For that, we perform a series of simulations of the problem and evaluate empirically the relationship between observed steady-state flow rates and hydraulic conductivity for a series of small applied suctions, as are used commonly in tension infiltration experiments.

2 Methodology

2.1 Principle of the proposed method

The most straightforward methodology to perform a k_u measurement is the unit-gradient experiment on a soil column, where the flow field is laterally confined by the column walls and gravity is the only force that acts on unidirectional water flow (*Dirksen*, 1999). Under perfect unit-gradient conditions, the steady flow rate in a soil sample is equal to the hydraulic conductivity according to Darcy's law. In such experiments, the base of a soil sample is confined by a porous disc, to which a stepwise changed suction h is applied. At the upper soil surface, infiltrating water is either provided by irrigation or through a second porous disk (double-disk or double-membrane method). In the first case, the infiltration rate and/or the suction at the lower boundary must be iteratively adjusted to ensure unit-gradient conditions. This is difficult and tedious to adjust manually (*Weller* et al., 2011) and may be further complicated by hysteresis and dynamic non-equilibrium effects (*Diamantopoulos* and *Durner*, 2012). In the second case, water can be applied by tension infiltration under the same suction as at the lower boundary (*Kramer* and *Meyer*, 1968; *Henseler* and *Renger*, 1969). A crucial point of the double-membrane method is ventilation of the soil, because without air access the soil water content cannot change with changing suction. By principle, the ventilation of soil in cores can be provided by lateral holes in the cylinders, but this induces some additional difficulties in the handling of the cores and excludes saturated percolation.

Our approach to solve the ventilation problem is to use a double-membrane apparatus where water infiltrates at the top of the soil column through a tension infiltrometer disk with a diameter smaller than the soil sample. This leaves part of the soil surface open and ensures a proper ventilation. Since the infiltration disk size is smaller than the open soil surface, the water flux trough the soil core will no longer be perfectly one-dimensional and strict unit-gradient conditions will not prevail in the whole soil domain. Consequently, the steady-state flow rate $q(h) = Q_{in}/A$, where Q_{in} is the volume of infiltrating water per time and A is the cross-sectional area of the soil column, will be smaller than the corresponding conductivity $k_u(h)$. This occurs despite equal pressure heads at the inflow and the outflow boundary, and is explained by a divergent flow field, as schematically illustrated in Fig. 1.

Expressing the ratio between flux rate and conductivity by a variable $f = q(h)/k_u(h)$ leads to:

$$k_u(h) = q(h)/f. \tag{1}$$

Thus, to evaluate measurements obtained with this experimental design, we need to know the relationship between the target variable [unsaturated soil hydraulic conductivity $k_u(h)$]

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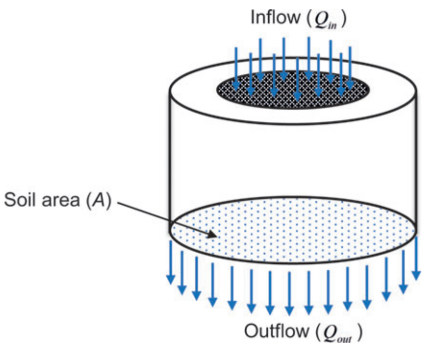

Figure 1: Schematic of inflow and outflow of water through the cylindrical soil sample in the proposed experiment with an area of the infiltration disk smaller than the area of the soil sample, *A*.

and the measured variables [suction *h* and rate of percolation through the soil sample, *q*(*h*)]. Specifically, the following research questions are of interest:

(1) How will *f* depend on the diameter *D* of the infiltration disc, which is smaller than the diameter of the soil surface?
(2) For a given infiltrometer disk radius, will *f*(*D*) be constant or vary with different suctions *h*?
(3) If we can find a unique relationship *f*(*D, h*), will this be dependent on soil texture, *i.e.*, on the specific shape of *k*(*h*)?

Our strategy to investigate these fundamental questions was creating "synthetic" experimental data with numerical modelling (section 2.2) and to evaluate them systematically. Based on the findings of the numerical study, we performed and evaluated real measurements, which will be documented in a companion paper (*Sarkar* et al., 2019).

2.2 Numerical modeling of the double-disk percolation

As indicated in section 2.1, a simple unit-gradient relationship cannot be directly applied in our experimental design, because the contact area of the tension infiltrometer is smaller than the total area of the soil cylinder in order to ensure free air access and ventilation of soil pores. To find a general relationship between the disk-size dependent percolation rate through a soil sample and the hydraulic conductivity of the soil, numerical investigations were carried out with the software package HYDRUS-2D/3D (*Šejna* and *Šimůnek*, 2007). HYDRUS-2D/3D is used for simulating water, heat, and solute movement in two- and three-dimensional variably saturated porous media. It solves the Richards equation numerically for transient saturated/unsaturated water flow with the finite element method.

The percolation was modelled as radial-symmetric flow problem, *i.e.*, the 3D flow process was solved as radial-symmetric 2D problem in radial coordinates, as illustrated in Fig. 2. The rectangular domain of radius r = 4 cm and depth z = 5 cm was discretised into a structured triangular finite element mesh with 81 nodes along r and 101 nodes along z. Hence, 16.000 triangular cells were created in the rectangular domain. To check if the numerical solution depends on resolution of the meshing, simulations for loam soil were studied additionally with coarser and finer meshing. The difference between the results obtained with the two different spatial resolutions was negligible.

The dependence of *f* on infiltrometer disk size, tension and soil type was investigated by performing series of "virtual experiments" for a soil sample of 8 cm diameter and 5 cm depth and the following conditions:

- The diameters of the disk of the tension infiltrometer at upper domain surface was varied: 4.5, 5.0, 6.0, 7.0, 8.0 cm.
- For each plate diameter, fluxes were calculated for different pressure heads *h* at the infiltration and outflow boundaries: 0, –0.5, –1, –2, –3 –4, –5, –7.5, –10 cm.
- For the smallest disk size, the dependency *f*(*h*) was evaluated for the 12 different soil classes of the USDA soil textural triangle (*USDA*, 2017), with hydraulic functions expressed by the van Genuchten–Mualem model (*van Genuchten*, 1980).
- Additionally, the dependency *f*(*h*) was evaluated for two soils with bimodal pore-size distributions, with hydraulic functions expressed by the *Durner* (1994) model.
- Finally, we investigated the dependency *f*(*h*) for the finest-textured soils with parameterizations, which avoid the artefact of the van Genuchten–Mualem conductivity function near saturation. This well-known artefact causes the hydraulic function to drop by orders of magnitudes right from saturation, which is not reflected in the retention curve and is non-physical.

The choice of the geometry data was determined by our physical experiments, where we used a diameter of the infiltrating disk of 4.5 cm in contact with a soil column with a diameter of

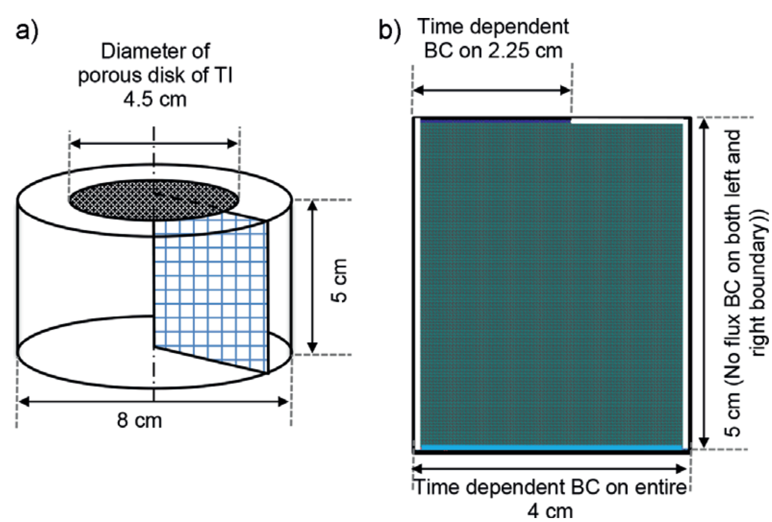

Figure 2: (a) Simulation domain in radial-symmetric coordinates with the 4.5 cm diameter porous disk of the tension infiltrometer (TI) at top, (b) 2D rectangular domain with time dependent suction head boundary conditions (blue line) applied on r = 2.25 cm at top and on entire r = 4 cm at bottom, and no flux conditions (white line) at the remaining boundaries.

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8.0 cm (*Sarkar* et al., 2019). Hydraulic parameters for all 12 soil classes of the USDA soil triangle were taken from *Carsel* and *Parrish* (1988), as shown in Tab. 1. The six parameters include saturated soil water content (θ_s), residual soil water content (θ_r), empirical parameters in the van Genuchten water retention model (α, n), saturated hydraulic conductivity (Ks) and a tortuosity parameter in hydraulic conductivity function (l). For the bimodal soils, the *Durner* (1994) model was used, which involves two sets of shape parameters (α_1, α_2, n_1, n_2), and a weighting parameter (w_1). The corresponding model parameters are listed in Tab. 2. For the three most fine-textured soils, we performed additional simulations with the alternative parameterization by *Vogel* et al. (2000), assuming a distinct air-entry of h = –2 cm, and with the model of *Iden* et al. (2015). The latter model uses the identical parameters as in the original formulation, but restricts the maximum pore-size in the conductivity model and thus avoids the artefact in the Mualem conductivity model. For further information about the models the reader is referred to the original literature.

The initial condition was provided in terms of pressure head, with a value of zero at the bottom and hydrostatic equilibrium from the lowest located nodal point. The boundary conditions (BC) were set as no flux BC on the left and right vertical boundary. At top and bottom boundary, identical pressure head conditions were applied that changed step-wise from no suction (h = 0 cm) to the highest suction (h = –10 cm) and *vice versa* with time. This is illustrated exemplarily in Fig. 3 for one of the real experiments (*Sarkar* et al., 2019). In the virtual experiments, the time steps in boundary conditions had different durations (5 to 50 minutes per step) for the twelve different soil classes, depending on their hydraulic properties and state of suction, in order to ensure that steady state conditions were always reached, which was checked by the equality of inflow and outflow rates to three significant digits, *i.e.*, $|(q_{in} - q_{out})/(0.5{\times}(q_{in} + q_{out}))| < 0.001$.

3 Results and discussion

First, the general relationship between the conductivity and flux, as defined by $f(D, h)$ in Eq. (1), was explored for a series of suction steps h and for varying diameters of the porous disk D of the tension infiltrometer. This part of the study was performed for the loam soil of the USDA soil texture triangle, with hydraulic parameters as listed in Tab. 1. In a second step, we quantified $f(h)$ for all twelve soil textures of the USDA texture triangle, assuming a disk diameter of D = 4.5 cm.

3.1 Effect of different diameters of the infiltration disk

The results of the numerical investigations for four out of the five different diameters of the porous disk, namely for 8 cm, 6 cm, 5 cm, and 4.5 cm are depicted in Fig. 4a–d. The figures depict the temporal evolution of inflow density (q_{in}), outflow density (q_{out}), boundary pressure heads (h), and the corresponding hydraulic conductivity $k(h)$ of the soil, which is taken from the hydraulic conductivity function. The boundary conditions changed in steps of 1 cm from initially –5 cm to 0 cm with step lengths of 10 minutes, shortened to 5 minutes near saturation.

Figure 4a–d illustrates two important findings. First, after each change of the pressure head, the outflow and the inflow densities differ for a short while, and then equilibrate again (in all cases in less than 5 minutes). The wetter the soil is, the faster is the equilibration. After a pressure increase (= reduction of suction), the inflow rate (blue line) is temporarily increased since the water pressure rises as compared to the previous steady flow situation, and gravity and the matric potential gradient act together in the same direction. At the bottom boundary, the opposite happens and the outflow is temporarily ceased. However, the flow rates at the boundaries quickly equilibrate and inflow and outflow rate

Table 1: Van Genuchten coefficients of the hydraulic properties of 12 different soil textural classes used in HYDRUS 2D/3D modelling.

Soil	θ_r	θ_s	α (1/cm)	n	k_s (cm d^{-1})	l
Sand	0.045	0.43	0.145	2.68	713	0.5
Loamy sand	0.057	0.41	0.124	2.28	350	0.5
Sandy loam	0.065	0.41	0.075	1.89	107	0.5
Loam	0.078	0.43	0.036	1.57	24	0.5
Silt	0.034	0.46	0.016	1.37	6	0.5
Silty loam	0.067	0.45	0.020	1.41	12	0.5
Sandy clay loam	0.100	0.39	0.059	1.48	32	0.5
Clay loam	0.095	0.41	0.019	1.31	5.76	0.5
Silty clay loam	0.089	0.43	0.010	1.23	1.44	0.5
Sandy clay	0.100	0.38	0.027	1.23	2.88	0.5
Silty clay	0.070	0.36	0.005	1.09	0.48	0.5
Clay	0.068	0.38	0.008	1.09	4.32	0.5

Table 2: Van Genuchten coefficients of the hydraulic functions of two bimodal soils.

Soil	θ_r	θ_s	α_1 (1/cm)	α_2 (1/cm)	n_1	n_2	w_1	k_s (cm d^{-1})	l
Aggregated Loam	0.160	0.663	0.3320	0.0112	3.12	2.527	0.65	344	0.5
Rideau Clay Loam	0.132	0.439	0.6643	0.0011	8.0	1.286	0.14	217	0.5

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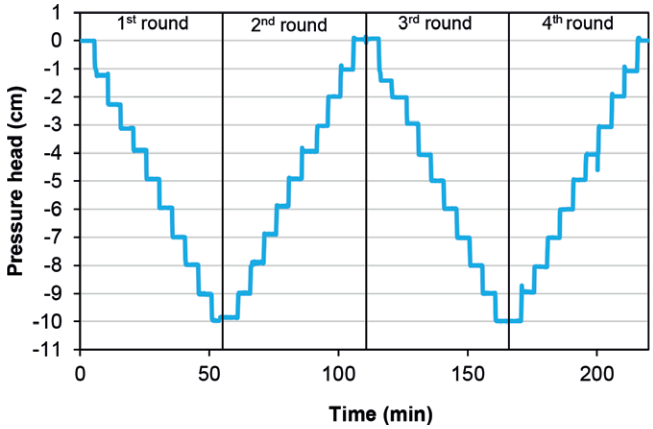

Figure 3: Typical example of time dependent boundary conditions: stepwise varying pressure head for four rounds (real data example for first sample of soil SAU from *Sarkar* et al., 2019).

converge to a new steady value. This basic pattern is independent of the size of the infiltration disk. During the time when outflow and inflow differ, the water content in the sample readjusts to an equilibrium water content that is determined by the soil water retention characteristic. Increasing suctions at the boundaries cause decreasing water contents and require an inflow of air into the sample. Decreasing suctions are associated with increasing water contents and require an escape of air.

The second finding concerns the difference between steady percolation rate and hydraulic conductivity (black dashed line), which is visible at all suctions and for all diameters except for the diameter of 8 cm. This is shown systematically in Fig. 5, which quantifies the dependence of f on pressure head and disk size. When the diameter of the infiltration disk matches exactly the diameter of the soil sample (= 8 cm),

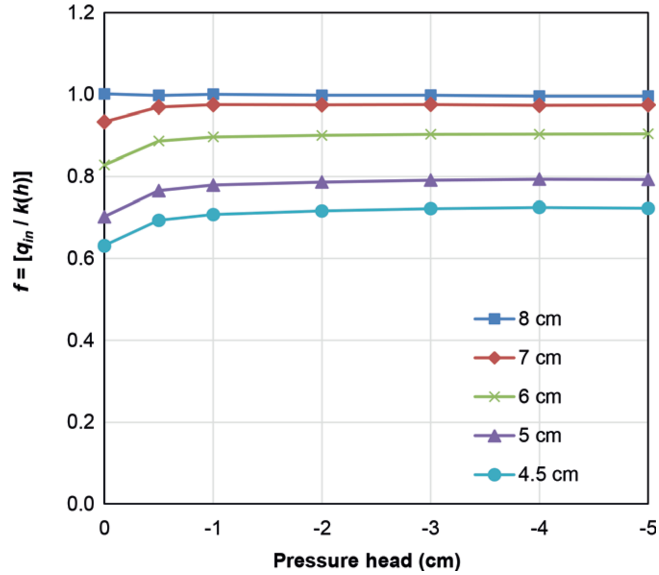

Figure 5: Variation of ratio f with pressure head for loam soil under different diameters of the tension infiltrometer disk.

then f = 1.0, *i.e.*, the observed steady-state percolation rate equals soil hydraulic conductivity, which reflects the standard unit-gradient condition. We include this banal observation because it confirms the accuracy of the numerical simulation.

With reduction in diameter, the steady-state percolation rate becomes smaller than the respective conductivity and the deviation increases for smaller diameters. For the smallest disk with diameter D = 4.5 cm, the observed steady percolation rate is approximately 72% of the soil hydraulic conductivity. We note that in this setting, the infiltration area covers only 32% of the total soil area. From Fig. 5 it becomes furthermore apparent that, for the loam, which is investigated here, $f(h)$ is

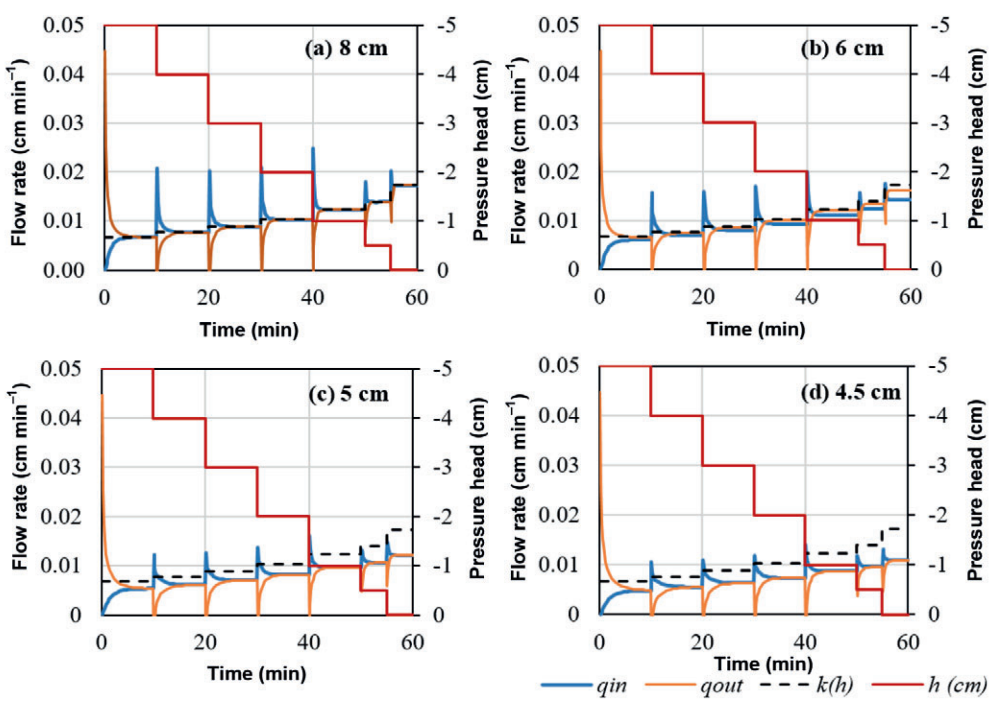

Figure 4: HYDRUS 2D/3D simulation results for loam soil: Variation of inflow and outflow rates under time dependent boundary conditions (stepwise increase of pressure head from –5 to 0 cm) and corresponding hydraulic conductivity for different diameters of porous disk of the tension infiltrometer. Note that the infiltration rate q_{in} is normalized with respect to the soil's cross-sectional area, *i.e.*, $q_{in} = Q_{in}/A_{soil}$.

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quite stable across the range of investigated suctions for any of the disk sizes. An exception to that occurs close to saturation, for pressure heads $h > -1$ cm.

3.2 Effect of soil texture

The influence of different soil textures was investigated for a disk size of D = 4.5 cm, because this size was used in the physical experiments. As described in section 2.2, twelve different soil classes from the soil textural triangle were used in the simulations. Figure 6a–d visualizes the temporal evolution of the relevant state variables exemplarily for four soil classes with strongly different textures (*i.e.*, sand, loam, silt and clay). Contrary to Fig. 4, we extended the pressure head range between saturation and –10 cm, to see whether the stable ratio that was found for the loam soil is also valid at greater suctions. Compared to the loam soil (Fig. 4), some general trends and similarities, but also some differences can be observed.

The first finding refers to the time which is required until inflow and outflow equilibrate. Obviously, the time to reach steady state equilibrium depends on texture. As intuitively expected, the higher the saturated hydraulic conductivity (*e.g.*, sand), the faster is the equilibration to steady state conditions. In accordance with that is the observation that it takes longer to reach steady state when the suctions are higher. As an example, the equilibration of inflow and outflow rate for sand needs 15 min at $h = -10$ cm, 12 min at $h = -7.5$ cm, about 6 min at –5 cm, and for the last change towards saturation it is less than 1 min.

For clay, a numerical problem becomes apparent in the last time step toward full saturation, *i.e.*, when the boundary condition is set to h = 0 cm. Here, simulated q_{in} and q_{out} show strong fluctuations, indicating numerical instability of the simulation. The reason for this lies in the shape of the parameterized van Genuchten/Mualem conductivity function. As evident from Fig. 7, which shows the van Genuchten–Mualem hydraulic conductivity functions of sand, loam, silt, and clay, the curve for clay changes sharply near saturation, with an almost vertical slope. This drastic change of hydraulic conductivity near saturation must be regarded as artefact of the van Genuchten–Mualem model. To get realistic results, the artefact should be removed, which can be done either by introducing an explicit air-entry value into the capillary saturation function (*Vogel* and *Cislerova*, 1988; *Ippisch* et al., 2006) or by introducing a maximum pore radius in the underlying pore-bundle model (*Iden* et al., 2015).

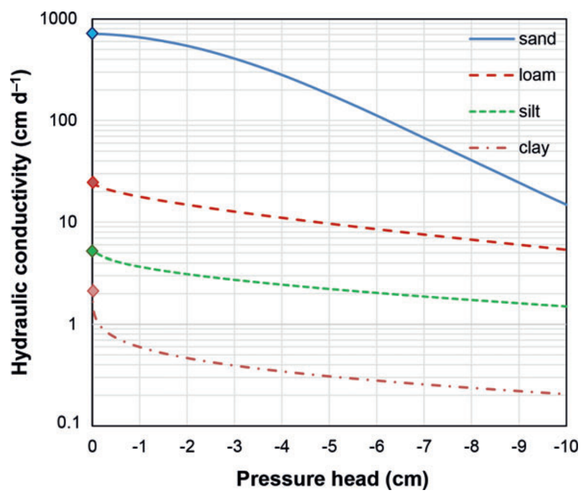

Figure 7: Hydraulic conductivity functions for sand, loam, silt and clay as given by the van Genuchten–Mualem model. The diamond points on the function indicate saturated hydraulic conductivities of respective soil texture.

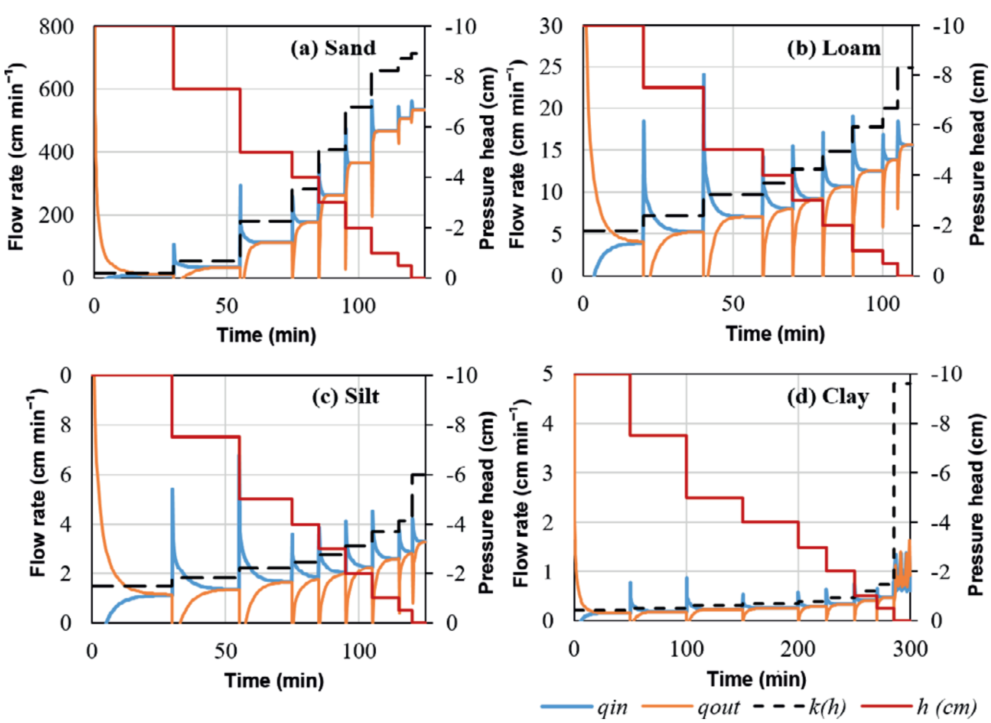

Figure 6: Variation of inflow and outflow rates in the HYDRUS simulation under time dependent BC (stepwise change of pressure head from –10 to 0 cm) and corresponding hydraulic conductivity for sand, loam, silt and clay, for an infiltration disk with 4.5 cm diameter.

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The results for $f(h)$ for all twelve texture classes are listed in Tab. 3 and depicted in Fig. 8. The trend toward a smaller value of f near saturation, which was observed for the loam, is not found for all soil classes. Specially, for coarse textured soils like sand, loamy sand, and sandy loam, f even increases toward saturation. The reason behind this is unknown. For the fine-textured soils, however, f decreases toward saturation, and this effect appears to be the more distinct, the finer the texture. The smallest ratio was found for clay with $f = 0.12$ at $h = 0$ cm. Clearly, this finding is correlated with the drastic change in hydraulic conductivity of clay near saturation (see Fig. 7). However, for the suction range 0.5 cm < h < 10 cm, the bandwidth of f for all investigated textures is remarkably small, with $0.6 < f < 0.75$, with a tendency of smaller f (around 0.65) for the sands compared to a larger value for medium or fine textured soils (0.72–0.74).

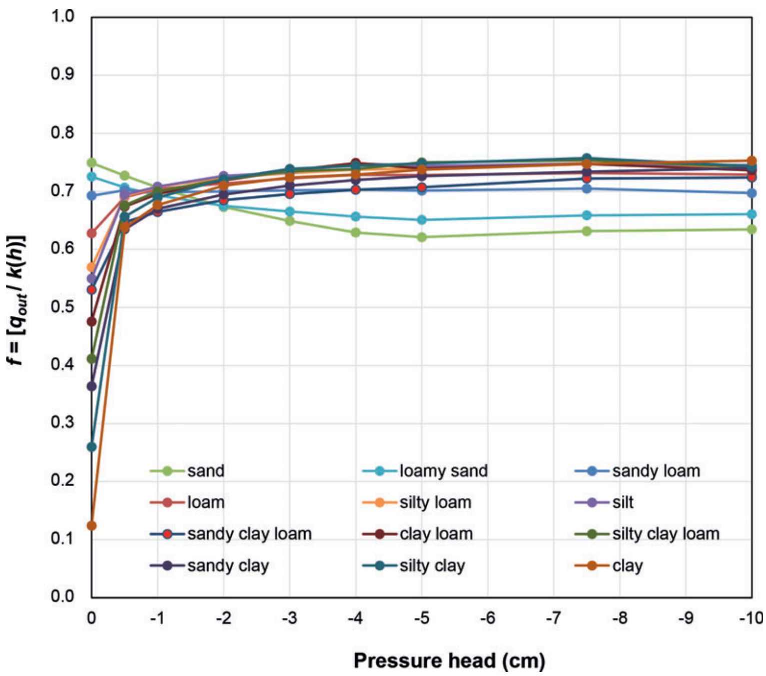

Figure 8: Variation of factor f with pressure head for twelve soil classes based on HYDRUS simulations with a 4.5 cm diameter infiltration disk.

Considering the numerous error sources and problems that are associated with unsaturated conductivity determinations and estimations in practical experiments, we believe that for the purpose of generalization, a representative value can be set to $f \approx 0.7$ for all soil texture classes in the suction range 0.5 cm < h < 10 cm, tolerating a relative error of about 10% (which is practically invisible on a log scale). This value of f will be used in the companion paper (*Sarkar* et al., 2019) where the proposed method for estimating unsaturated hydraulic conductivities is used in real experiments.

3.3 Identification of k_u for fine-texture soils avoiding the steep conductivity drop near saturation

Figure 9 depicts the three investigated cases of the parametrization of the conductivity curve on the factor f for silty clay as example soil. The results for pure clay and silty clay loam are qualitatively almost identical, and are not shown here for space reasons. The van Genuchten–Mualem parameterized conductivity function, indicated as "VGM", drops right at saturation by about an order of magnitude. This is not reflected in the retention curve (not shown here, because it shows no drop between saturation and $h = -10$ hPa). This sharp drop is a well-known artefact of the VGM model conceptualization, caused by an infinite steepness of the conductivity function at saturation, as extensively discussed by *van Genuchten* and *Nielsen* (1985), *Vogel* et al. (2000), *Ippisch* et al. (2006), and *Iden* et al. (2015). The VGM function leads to a value of f which is near saturation far out of the narrow range of $0.6 < f < 0.75$, which was observed for the coarser soil types, where this artefact does not occur (Fig. 8). Figure 9 (left) shows how alternative parameterizations lead to considerably different conductivity functions with zero slope (*Vogel* et al.,

Table 3: Calculated f-values for 12 different soil classes for different negative pressure heads.

h (cm)	Sand	Loamy sand	Sandy loam	Loam	Sandy clay loam	Silty loam	Silt	Clay loam	Silty clay loam	Sandy clay	Silty clay	Clay
–10	0.63	0.66	0.70	0.73	0.72	0.74	0.75	0.74	0.74	0.74	0.74	0.75
–7.5	0.63	0.66	0.70	0.73	0.72	0.75	0.75	0.75	0.75	0.73	0.76	0.75
–5	0.62	0.65	0.70	0.73	0.71	0.74	0.74	0.74	0.75	0.73	0.75	0.74
–4	0.63	0.66	0.70	0.73	0.70	0.74	0.75	0.75	0.74	0.72	0.74	0.73
–3	0.65	0.67	0.70	0.72	0.70	0.73	0.73	0.74	0.73	0.71	0.74	0.72
–2	0.67	0.67	0.70	0.71	0.68	0.72	0.73	0.72	0.72	0.69	0.72	0.71
–1	0.71	0.69	0.70	0.70	0.66	0.71	0.71	0.70	0.70	0.67	0.69	0.68
–0.5	0.73	0.71	0.70	0.69	0.65	0.69	0.69	0.67	0.68	0.63	0.66	0.64
0	0.75	0.73	0.69	0.63	0.53	0.57	0.55	0.48	0.41	0.36	0.26	0.12

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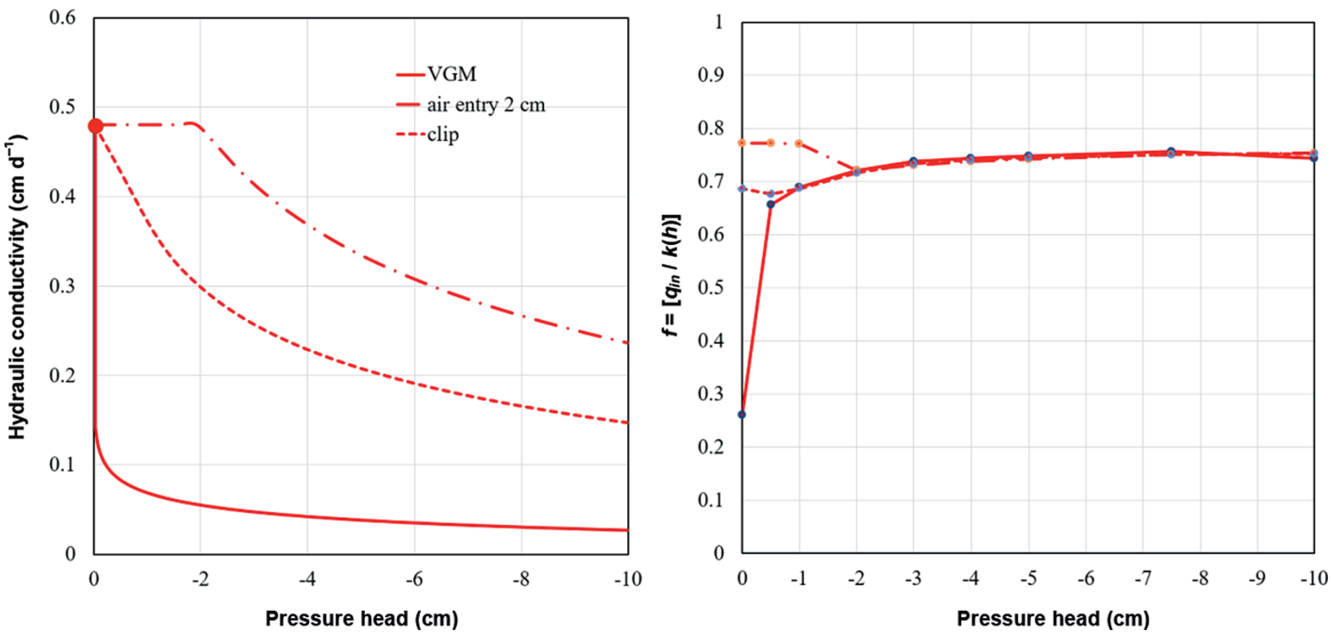

Figure 9: Shape of the hydraulic conductivity curve near saturation (left) and its effect on the identified ratio *f* for material "silty clay" (right). "GM", "air entry 2 cm", and "hclip" indicate parametrizations according to *van Genuchten* (1980), *Vogel* et al. (2000), and *Iden* et al. (2015), respectively.

2000) or finite slope (*Iden* et al., 2015) at saturation. Our simulations with these parameterized functions show that the extremely low *f* values near saturation of the MVG model now disappear [Fig. 9 (right), Tab. 4]. Actually, the *f* values lie now within the expected range across the full pressure head range.

3.4 Identification of k_u for soils with bimodal pore-size distributions

A specific need for k_u measurements near saturation lies in the identification of the effects of aggregation on the conductivity function. This can be expressed by bimodal effective hydraulic properties (*Durner*, 1994). To investigate the identification of conductivity functions of such soils, we simulated the percolation method for two soils from the literature, with rather extreme properties as shown in Fig. 10. Data for "Aggregated Loam" are taken from *Smettem* and *Kirkby* (1990), whereas data for "Rideau Clay Loam" are taken from *De Jong* et al. (1992). The Aggregated Loam consists of a repacked sample with many small aggregates, which leads to a package with distinct and relatively voluminous inter-aggregated pore-space, which drains in the suction range between 1 and 30 cm water column. Rideau Clay Loam, on the other side, is characterized by a rather coherent clay matrix which is pervaded by macropores, which take only few percent of pore space volume, but reduce the conductivity function drastically by four orders of magnitude in the suction range 1 to 5 cm water column (Fig. 10).

Figure 11 and Tab. 4 indicate that the ratio between flux density and conductivity for the proposed steady-state percolation is somewhat different from the unimodal soils. Specifically, *f* is lower in the range where the conductivity function drops most steeply. For Rideau Clay Loam, the ratio *f* reaches a minimum value of 0.37 at *h* = –2 cm. This means that the assumption of *f* = 0.7 would give, at this suction, an underestimation of the estimated true conductivity of about 45%. This might appear to be a considerable error on first sight, but it is quite minor, if we consider the actual change of the function of about four orders of magnitude in the pressure head range between 1 and 5 hPa. Furthermore, this error reduces quite quickly for higher suctions (Fig. 11).

Table 4: Calculated *f* values for the silty clay soil with alternative conductivity parametrizations and the two bimodal soils.

h (cm)	Silty clay (VGM)	Silty clay (air entry 2 cm)	Silty clay (Iden et al., 2015)	Aggregated Loam	Rideau Clay Loam
–10	0.74	0.75	0.75	0.70	0.77
–7.5	0.76	0.75	0.75	0.66	0.77
–5	0.75	0.74	0.74	0.57	0.76
–4	0.74	0.73	0.74	0.51	0.73
–3	0.74	0.73	0.73	0.49	0.47
–2	0.72	0.72	0.71	0.49	0.37
–1	0.69	0.77	0.69	0.57	0.43
–0.5	0.66	0.77	0.68	0.64	0.60
0	0.26	0.77	0.68	0.72	0.73

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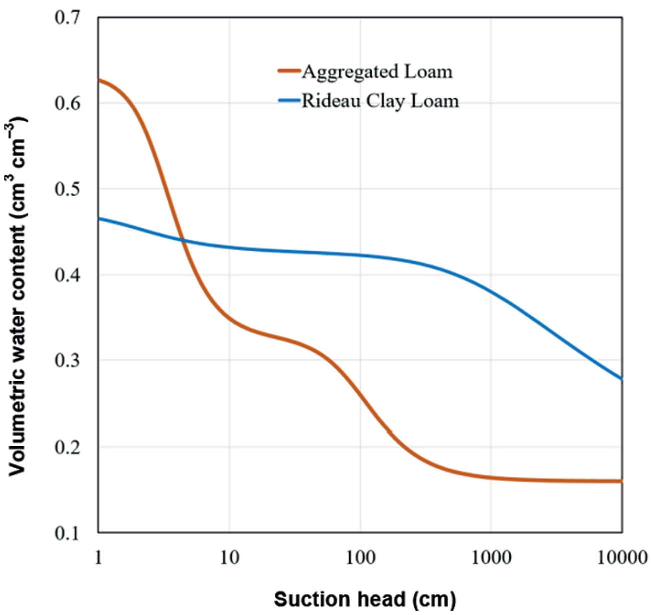

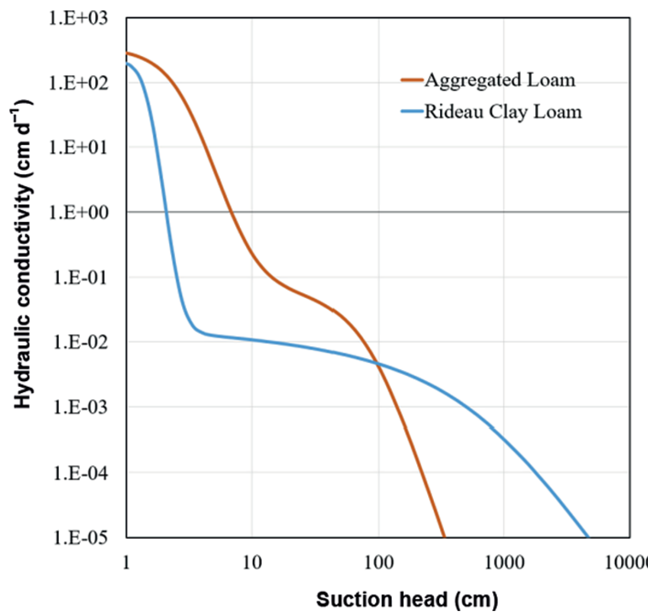

Figure 10: Retention curves (left) and conductivity curves (right) for two bimodal soils: Aggregated Loam (*Smettem* and *Kirkby*, 1990) and Rideau Clay Loam (*De Jong* et al., 1992). Note that functions are shown on a log scale and thus are plotted *vs.* the positive suction head.

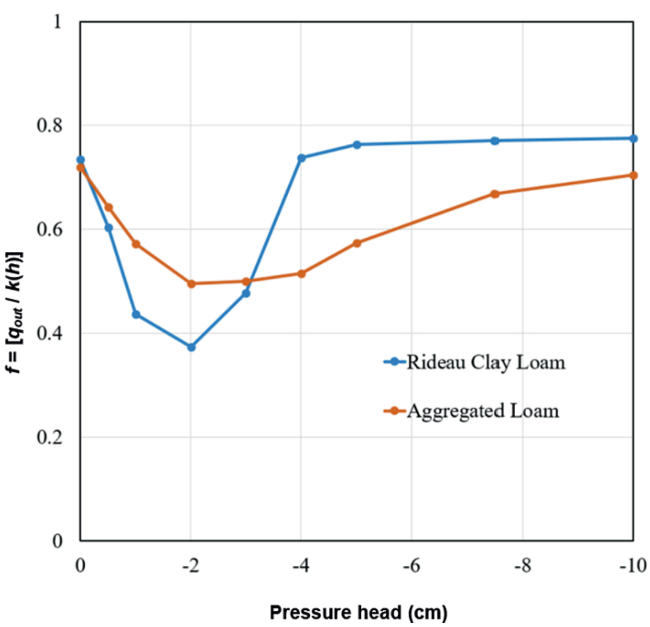

Figure 11: Identified ratio f for materials "Rideau Clay Loam" and "Aggregated Loam".

4 Summary and conclusions

Percolation experiments on soil columns under unit-gradient conditions at various constant suctions are the most straightforward way to measure unsaturated hydraulic conductivity near saturation (*Dirksen*, 1999) and are needed to validate or falsify indirect estimation methods. Due the experimental difficulty and the non-availability of commercial apparatus, they are, however, rarely performed in the lab. The application of water by tension infiltrometry through a porous plate is easy to construct, but problematic, if the disk covers the whole surface are of the soil or if it is tightly fitted to the sample's wall, because this does not allow a proper ventilation of the soil, which is needed to replace water by air when the suction is successively increased.

To solve this problem, we have investigated in this paper a variant of the unit-gradient method, in which the area of the tension infiltrometer is reduced as compared to the cross-sectional area of the soil sample. By numerical simulations with HYDRUS 2D/3D, we could show that a clear relationship exists between the size of the infiltration disk and the observed steady-state percolation rate through the soil. Furthermore, the simulations showed that for a given disk size, the ratio between percolation rate q in the soil sample and unsaturated conductivity k_u is almost constant for a range of suctions h and only weakly dependent on soil type. At saturation (*i.e.*, $h = 0$ cm), the ratio $f = q(h)/k_u(h)$ is quite different for different soil types (Fig. 8). However, beyond a certain suction, f approaches a relatively constant value, which is similar for all soil types. For a soil sample size as used in the commercial devices KSAT and HYPROP (5 cm high, 8 cm wide, 250 cm^3) and a diameter of an infiltration disk of 4.5 cm as used in the commercial Mini Disk Infiltrometer, we could show that the steady-state flow rate is approximately $f \approx 0.7$ times the unsaturated conductivity at suctions > 0.5 cm. Accordingly, calculating unsaturated hydraulic conductivity from this type of steady-state percolation experiments should give results with a relative error that is smaller than 10%. This is also true for the very fine textured soils, if parameterizations are used for the conductivity function, which avoid the non-physical steep drop of the VGM conductivity function right at saturation. For distinct bimodal soils, the relative error can become larger, reaching up to 45% in a narrow pressure head range where the conductivity function drops most steeply. Considering that the drop of the conductivity function in that pressure head range is orders of magnitudes, this local error still seems to be tolerable.

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The method has thus the potential to bridge the gap between the results of established methods for saturated conductivity (*e.g*., percolation with the falling head method) and for unsaturated conductivity (*e.g*., the evaporation method) in the medium dry range. The uncertainty about the shape of a fitted parametric hydraulic conductivity function in the important range between saturation and medium dry conditions is greatly reduced.

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