# Hamiltonian Cycles

# Hamiltonian Cycle

- A *Hamiltonian cycle* is a spanning cycle in a graph.
  - The *circumference* of a graph is the length of its longest cycle.
  - A *Hamiltonian path* is a spanning path.
  - A graph with a spanning cycle is a *Hamiltonian graph*.

# Necessary and Sufficient Conditions

- [Necessary:] If G has a Hamiltonian cycle, then for any set $S \subseteq V$, the graph G–S has at most |S| components.

- [Sufficient: *Dirac:1952*] If G is a simple graph with at least three vertices and $\delta(G) \geq n(G)/2$, then G is Hamiltonian.

- [Necessary and sufficient:] If G is a simple graph and *u,v* are distinct non-adjacent vertices of G with $d(u) + d(v) \geq n(G)$, then G is Hamiltonian if and only if G + *uv* is Hamiltonian.

# Hamiltonian Closure

The Hamiltonian closure of a graph G, denote C(G), is the supergraph of G on V(G) obtained by iteratively adding edges between pairs of non-adjacent vertices whose degree sum is at least *n*, until no such pair remains.

- The closure of G is well-defined
- A simple *n*-vertex graph is Hamiltonian if and only if its closure is Hamiltonian

# And more…

- If $\chi(G) \geq \alpha(G)$, then G has a Hamiltonian cycle (unless G = $K_2$)