Euler Graphs and Digraphs

Euler Circuit

- We use the term *circuit* as another name for *closed trail*.
  - A circuit containing every edge of G is an *Eulerian circuit.*
  - A graph whose edges comprise a single closed trail is *Eulerian.*

Properties

- Non-trivial maximal trails in even graphs are closed.
- A finite graph G is Eulerian if and only if all its vertex degrees are even and all its edges belong to a single component.
- For a connected nontrivial graph with $2k$ odd vertices, the minimum number of pairwise edge-disjoint trails covering the edges is $\max\{k, 1\}$.

Fleury's Algorithm

Input: A graph G with one non-trivial component and at most two odd vertices.

Initialization: Start at a vertex that has odd degree unless G is even, in which case start at any vertex.

Iteration: From the current vertex, traverse any remaining edge whose deletion from the graph does not leave a graph with two non-trivial components. Stop when all edges have been traversed.

Euler Trails in Directed Graphs

Input: A digraph G that is an orientation of a connected graph and has $d^+(u) = d^-(u)$ for all $u \in V(G)$.

Step1: Choose a vertex $v \in V(G)$. Let G′ be the digraph obtained from G by reversing direction on each edge. Search G′ to construct T′ consisting of paths from *v* to all other vertices.

Step2: Let T be the reversal of T′. T contains a *u,v*-path in G for each $u \in V(G)$. Specify an arbitrary ordering of the edges that leave each vertex *u*, except that for $u \neq v$, the edge leaving *u* in T must come last.

Step3: Construct an Eulerian circuit from *v* as follows. Whenever *u* is the current vertex, exit along the next unused edge in the ordering specified for edges leaving *u*.

The Chinese Postman Problem

- Suppose a mail carrier traverses all edges in a road network, starting and ending at the same vertex.
  - The edges have non-negative weights representing distance or time.
  - We seek a closed walk of minimum total length that uses all the edges.