Degree Sequences & Digraphs

Degree Sequence

- The *degree sequence* of a graph is the list of vertex degrees, usually written in non-increasing order, as $d_1 \geq \ldots \geq d_n$

Algorithmic or Constructive Proofs

- Every loop-less graph G has a bipartite sub-graph with at least e(G)/2 edges
- The non-negative integers, $d_1 \geq \ldots \geq d_n$ are the vertex degrees of some graph if and only if $\Sigma\, d_i$ is even

Graphic Sequence

- A *graphic sequence* is a list of non-negative numbers that is the degree sequence of some simple graph.
  – A simple graph with degree sequence *d* *realizes* *d*.

Graphic: necessary & sufficient

- For $n>1$, the non-negative integer list *d* of size *n* is graphic if and only if d' is graphic, where d' is the list of size *n-1* obtained from *d* by deleting its largest element Δ, and subtracting 1 from its Δ next largest elements.

[Havel 1955, Hakimi 1962]

2-switch

A *2-switch* is a replacement of a pair edges *xy* and *zw* in a simple graph by the edges *yz* and *wx*, given that *yz* and *wx* did not appear in the graph originally.

- If G and H are two simple graphs with vertex set V, $d_G(v) = d_H(v)$ for every $v \in V$ if and only if there is a sequence of 2-switches that transforms G into H.

[Berge 1973]

Orientation of a Digraph

- An *orientation* of a graph G is a digraph D obtained from G by choosing an orientation ($x \rightarrow y$ or $y \rightarrow x$) for each edge $xy \in E(G)$.
- A *tournament* is an orientation of a complete graph.

King

- A *king* is a vertex from which every vertex is reachable by a path of length at most 2.

- Every tournament has a king.