Graph Theory: Trees

Pallab Dasgupta,

Professor, Dept. of Computer Sc. and Engineering, IIT Kharagpur

pallab@cse.iitkgp.ernet.in

Trees and Spanning Trees

- A graph having no cycles is *acyclic.*
- A *forest* is an acyclic graph.
- A *leaf* is a vertex of degree 1.
- A *spanning sub-graph* of G is a sub-graph with vertex set V(G).
- A *spanning tree* is a spanning sub-graph that is a tree.

Distances

- If G has a *u,v*-path, then the distance from *u* to *v*, written $d_G(u,v)$ or simply *d(u,v)*, is the least length of a *u,v*-path.
  - If G has no such path, then $d(u,v) = \propto$

Tree: Characterization

- An n-vertex graph G (with $n \geq 1$) is a tree iff:
  - G is connected and has no cycles
  - G is connected and has $n-1$ edges
  - G has $n-1$ edges and no cycles
  - For $u,v \in V(G)$, G has exactly one *u,v*-path

Some results …

- Every tree with at least two vertices has at least two leaves.
  - Deleting a leaf from a tree with *n* vertices produces a tree with *n-1* vertices.

- If T is a tree with *k* edges and G is a simple graph with $\delta(G) \geq k$, then T is a sub-graph of G.

Some results …

- If T and T′ are two spanning trees of a connected graph G and e $\in E(T) - E(T')$, then there is an edge e′ $\in E(T') - E(T)$ such that $T - e + e'$ is a spanning tree of G.

Diameter and Radius

- The *eccentricity* of a vertex *u*, written $\varepsilon(u)$, is the maximum of its distances to other vertices.

- In a graph G, the *diameter*, diamG, and the *radius*, radG, are the maximum and minimum of the vertex eccentricities respectively.

- The *center* of G is the subgraph induced by the vertices of minimum eccentricity.

Counting Trees

- **There are n^{n-2} trees with vertex set [n].**

Prüfer Code / Sequence

Algorithm: *Production of $f(T) = \{a_1, \ldots, a_{n-2}\}$*

Input: A tree T with vertex set $S \subseteq \aleph$.

Iteration: At the i^{th} step, delete the least remaining leaf, and let a_i be the *neighbor* of this leaf.