

Graph Theory: Euler Graphs and Digraphs



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Euler Circuit

- We use the term *circuit* as another name for *closed trail*.
 - A circuit containing every edge of G is an *Eulerian circuit*.
 - A graph whose edges comprise a single closed trail is *Eulerian*.

Properties

- **Non-trivial maximal trails in even graphs are closed.**
- **A finite graph G is Eulerian if and only if all its vertex degrees are even and all its edges belong to a single component.**
- **For a connected nontrivial graph with $2k$ odd vertices, the minimum number of pairwise edge-disjoint trails covering the edges is $\max\{k, 1\}$.**

Fleury's Algorithm

Input: A graph G with one non-trivial component and at most two odd vertices.

Initialization: Start at a vertex that has odd degree unless G is even, in which case start at any vertex.

Iteration:

From the current vertex, traverse any remaining edge whose deletion from the graph does not leave a graph with two non-trivial components. Stop when all edges have been traversed.

Euler Trails in Directed Graphs

- Input:** A digraph G that is an orientation of a connected graph and has $d^+(u) = d^-(u)$ for all $u \in V(G)$.
- Step1:** Choose a vertex $v \in V(G)$. Let G' be the digraph obtained from G by reversing direction on each edge. Search G' to construct T' consisting of paths from v to all other vertices.
- Step2:** Let T be the reversal of T' . T contains a u, v -path in G for each $u \in V(G)$. Specify an arbitrary ordering of the edges that leave each vertex u , except that for $u \neq v$, the edge leaving u in T must come last.
- Step3:** Construct an Eulerian circuit from v as follows. Whenever u is the current vertex, exit along the next unused edge in the ordering specified for edges leaving u .

The Chinese Postman Problem

- **Suppose a mail carrier traverses all edges in a road network, starting and ending at the same vertex.**
 - **The edges have non-negative weights representing distance or time.**
 - **We seek a closed walk of minimum total length that uses all the edges.**