Graph Theory: Proof Techniques

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Induction

- **If *u* and *v* are distinct vertices in G, then every *u,v-walk* in G contains a *u,v-path.***
- **Every closed odd walk contains an odd cycle.**

Contra-positive

We use: $(\neg B \Rightarrow \neg A) \equiv (A \Rightarrow B)$

- A graph is connected iff for every partition of its vertices into two non-empty sets, there is an edge with endpoints in both sets
- An edge of a graph is a cut-edge iff it belongs to no cycle

Contradiction

We prove $A \Rightarrow B$ by showing that "A true and B false" is impossible

- **Suppose G has a vertex set $\{v_1, \ldots, v_n\}$, with $n \geq 3$. If at least two of the sub-graphs from $G-v_1, \ldots, G-v_n$ are connected, then G is connected**

- **A graph is bipartite iff it has no odd cycle**

Extremality

- If G is a simple graph in which every vertex degree is at least *k*, then G contains a path of length at least *k*.
  - If $k \geq 2$, then G also contains a cycle of length at least $k+1$.
- If G is a nontrivial graph and has no cycle, then G has a vertex of degree 1.
- Every nontrivial graph has at least two vertices that are not cut vertices.

The Reconstruction Conjecture

- **A graph G is *reconstructible* if we can reconstruct G from the list $\{ G - v_i : v_i \in V(G) \}$**

- **The famous *unsolved* reconstruction conjecture says that every graph with at least three vertices is reconstructible.**