First Order Logic

Course: CS40002

Instructor: Dr. Pallab Dasgupta

Department of Computer Science & Engineering

Indian Institute of Technology Kharagpur

Knowledge and Reasoning

- Representation, Reasoning and Logic
- Propositional Logic
- First-Order Logic
- Inference in first-order logic

First-order Logic

- Constant →
  A | 5 | Kolkata | …
- Variable →
  a | x | s | …
- Predicate →
  Before | HasColor | Raining | …
- Function →
  Mother | Cosine | Headoflist | …

First-order Logic

- Sentence → AtomicSentence
  | Sentence Connective Sentence
  | Quantifier Variable, … Sentence
  | ¬ Sentence | (Sentence)
- AtomicSentence →
  Predicate(Term, …) | Term = Term
- Term →
  Function(Term, …) | Constant | Variable
- Connective → $\Rightarrow$ | $\wedge$ | $\vee$ | $\Leftrightarrow$
- Quantifier → $\forall$ | $\exists$

Examples

- Not all students take both History & Biology
- Only one student failed History
- Only one student failed both History & Biology
- The best score in History is better than the best score in Biology
- No person likes a professor unless the professor is smart
- Politicians can fool some of the people all the time, and they can fool all the people some of the time, but they cant fool all the people all the time

Examples

- Russel's Paradox:
  - There is a single barber in town.
  - Those and only those who do not shave themselves are shaved by the barber.
  - Who shaves the barber?

Inference rules

- Universal elimination:
  - $\forall x$ Likes(x, IceCream) with the substitution {x / Einstein} gives us Likes(Einstein, IceCream)
  - The substitution has to be done by a ground term

Inference rules

- Existential elimination:
  - From $\exists x$ Likes(x, IceCream) we may infer Likes(Man, IceCream) as long as Man does not appear elsewhere in the Knowledge base
- Existential introduction:
  - From Likes(Monalisa, IceCream) we can infer $\exists x$ Likes(x, IceCream)

Reasoning in first-order logic

- The law says that it is a crime for a Gaul to sell potion formulas to hostile nations.
- The country Rome, an enemy of Gaul, has acquired some potion formulas, and all of its formulas were sold to it by Druid Traitorix.
- Traitorix is a Gaul.
- Is Traitorix a criminal?

Generalized Modus Ponens

- For atomic sentences p_i, p_i', and q, where there is a substitution θ such that SUBST(θ, p_i') = SUBST(θ, p_i), for all i:

$$\frac{p_1', p_2', \ldots, p_n', (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{SUBST(\theta, q)}$$

Unification

UNIFY(p,q) = θ where SUBST(θ,p) = SUBST(θ,q)

Examples:

UNIFY(Knows(Erdos, x),Knows(Erdos, Godel))
= {x / Godel}

UNIFY(Knows(Erdos, x), Knows(y,Godel))
= {x/Godel, y/Erdos}

Unification

UNIFY(p,q) = θ where SUBST(θ,p) = SUBST(θ,q)

Examples:

UNIFY(Knows(Erdos, x), Knows(y, Father(y)))
= { y/Erdos, x/Father(Erdos) }

UNIFY(Knows(Erdos, x), Knows(x, Godel)) = F

We require the most general unifier

Reasoning with Horn Logic

- We can convert Horn sentences to a canonical form and then use generalized Modus Ponens with unification.
  - We skolemize existential formulas and remove the universal ones
  - This gives us a conjunction of clauses, that are inserted in the KB
  - Modus Ponens help us in inferring new clauses
- Forward and backward chaining

Completeness issues

- Reasoning with Modus Ponens is incomplete
- Consider the example –

$\forall x\ P(x) \Rightarrow Q(x)$

$\forall x\ Q(x) \Rightarrow S(x)$

$\forall x\ \neg P(x) \Rightarrow R(x)$

$\forall x\ R(x) \Rightarrow S(x)$

- We should be able to conclude S(A)
- The problem is that $\forall x\ \neg P(x) \Rightarrow R(x)$ cannot be converted to Horn form, and thus cannot be used by Modus Ponens

Godel’s Completeness Theorem

- For first-order logic, any sentence that is entailed by another set of sentences can be proved from that set
  - Godel did not suggest a proof procedure
  - In 1965 Robinson published his resolution algorithm

- **Entailment in first-order logic is semi-decidable, that is, we can show that sentences follow from premises if they do, but we cannot always show if they do not.**

The validity problem of first-order logic

- [Church] The validity problem of the first-order predicate calculus is partially solvable.
- Consider the following formula:

$$[\bigwedge_{i=1}^{n} p(f_i(a), g_i(a))$$

$$\wedge \forall x \forall y [p(x, y) \Rightarrow \bigwedge_{i=1}^{n} p(f_i(x), g_i(x))]]$$

$$\Rightarrow \exists z \; p(z, z)$$