Learning:Neural Networks

Course: CS40022

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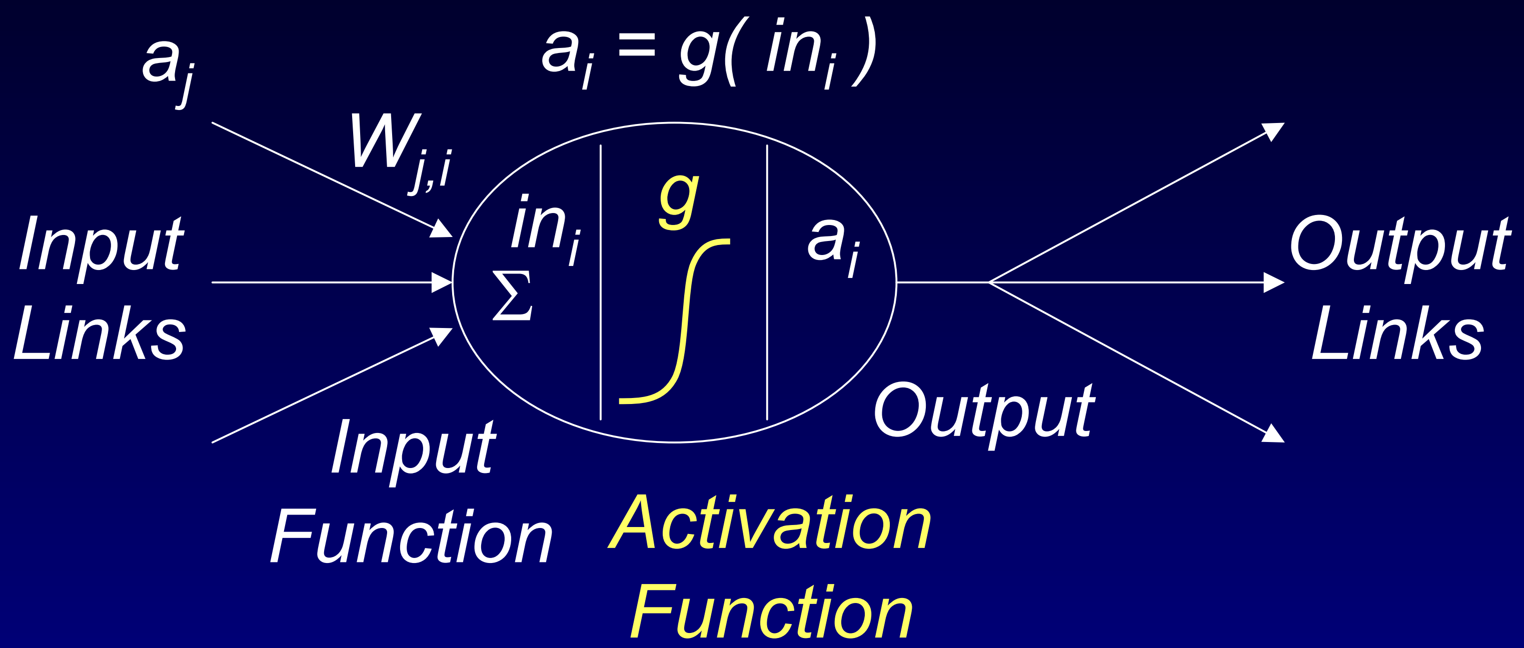
Neural Networks

- A neural network consists of a set of nodes (neurons/units) connected by links
  - Each link has a numeric weight

- Each unit has:
  - a set of input links from other units,
  - a set of output links to other units,
  - a current activation level, and
  - an activation function to compute the activation level in the next time step.

Basic definitions



Basic definitions

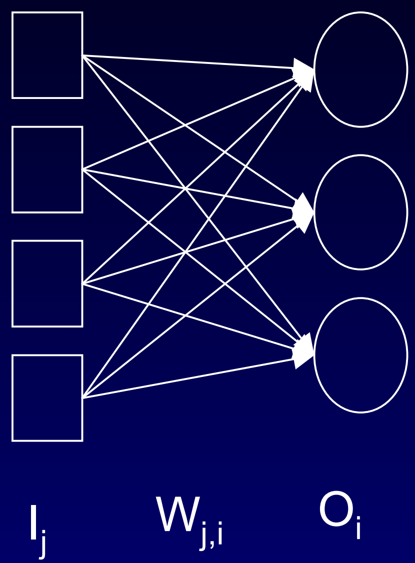
- The total weighted input is the sum of the input activations times their respective weights:

$$in_i = \sum_j W_{j,i} a_j$$

- In each step, we compute:

$$a_i \leftarrow g(in_i) = g\left(\sum_j W_{j,i} a_j \right)$$

Learning in Single Layer Networks



- If the output for a output unit is O, and the correct output should be T, then the error is given by:

  $$Err = T - O$$

- The weight adjustment rule is:

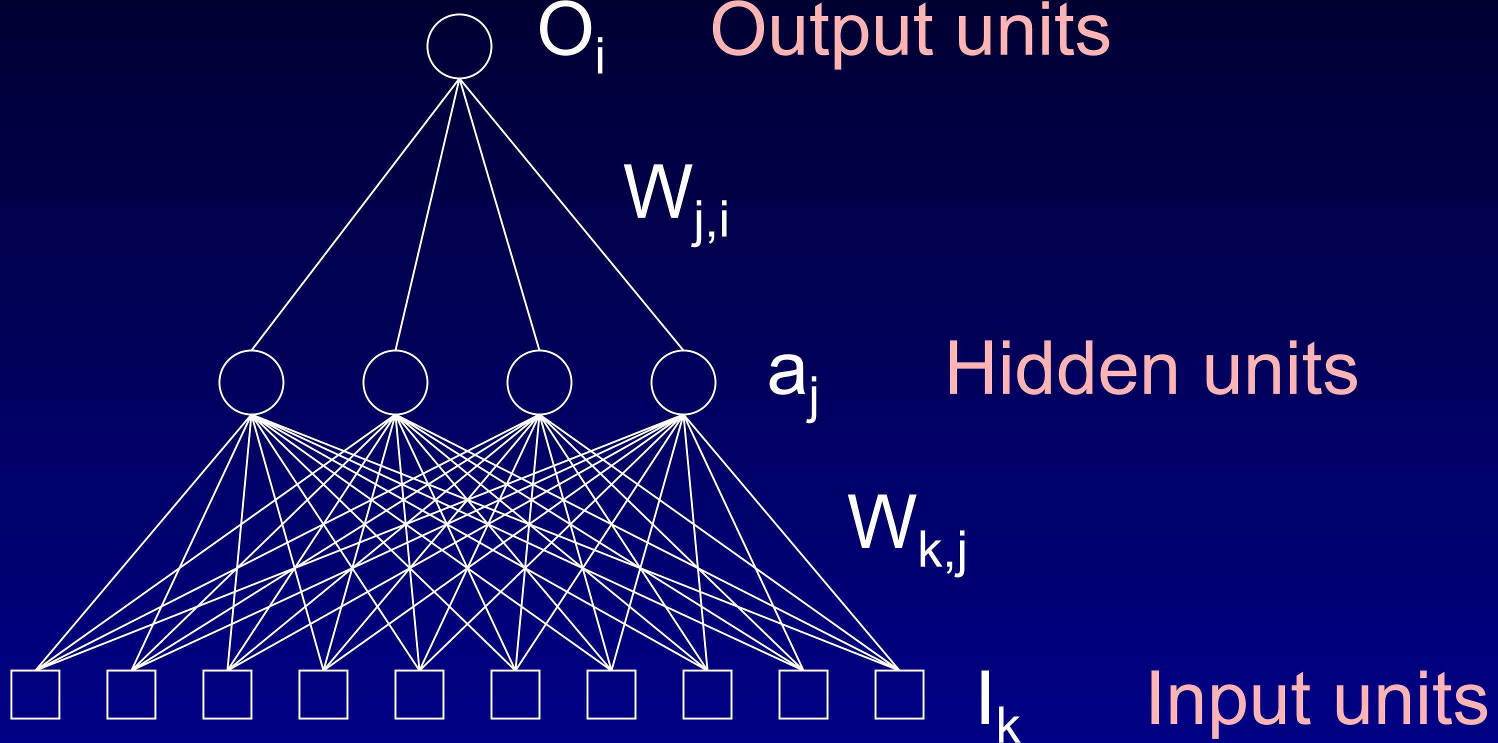
  $$W_j \leftarrow W_j + \alpha \times I_j \times Err$$

  where α is a constant called the learning rate

Learning in Single Layer Networks

- This method is unable to learn all types of functions
  - can learn only linearly separable functions
- Used in competitive learning

Two-layer Feed-Forward Network



Back-Propagation Learning

- $Err_i = (T_i - O_i)$ is the error at the output node
- The weight update rule for the link from unit j to output unit i is:

$$W_{j,i} \leftarrow W_{j,i} + \alpha \times a_j \times Err_i \times g'(in_i)$$

where g' is the derivative of the activation function g.

Back-Propagation Learning

- Let Δ_i = Err_i $g'(in_i)$. Then we have:

$$W_{j,i} \leftarrow W_{j,i} + \alpha \times a_j \times \Delta_i$$

Error Back-Propagation

- For updating the connections between the inputs and the hidden units, we need to define a quantity analogous to the error term for the output nodes
  - Hidden node j is responsible for some fraction of the error Δ_i each of the output nodes to which it connects

Error Back-Propagation

- So, the Δ_i values are divided according to the strength of the connection between the hidden node and the output node, and propagated back to provide the Δ_j values for the hidden layer

$$\Delta_j = g'(in_j)\sum_i W_{j,i}\Delta_i$$

Error Back-Propagation

- The weight update rule for weights between the inputs and the hidden layer is:

$$W_{k,j} \leftarrow W_{k,j} + \alpha \times I_k \times \Delta_j$$

The theory

- The total error is given by:

$$E = \frac{1}{2}\sum_i (T_i - O_i)^2 \quad = \quad \frac{1}{2}\sum_i \left(T_i - g\left(\sum_j W_{j,i} a_j \right) \right)^2$$

$$= \frac{1}{2}\sum_i \left(T_i - g\left(\sum_j W_{j,i} \left(\sum_k W_{k,j} I_k \right) \right) \right)^2$$

The theory

- Only one of the terms in the summation over i and j depends on a particular $W_{j,i}$, so all other terms are treated as constants with respect to $W_{j,i}$

$$\frac{\partial E}{\partial W_{j,i}} = -a_j (T_i - O_i) g'\left(\sum_j W_{j,i} a_j\right) = -a_j (T_i - O_i) g'(in_i) = -a_j \Delta_i$$

Similarly: $$\frac{\partial E}{\partial W_{k,j}} = -I_k \Delta_j$$