

The following is the computation of the probabilities of several different event combinations of the Burglary-Alarm belief network example given in Russell-Norvig. The ones marked in blue have been mentioned in the book, but not derived. The ones in black have been used to derive the ones marked in blue.

$$\begin{aligned} P(B) &= 0.001 \\ P(B') &= 1 - P(B) = 0.999 \end{aligned}$$

$$\begin{aligned} P(E) &= 0.002 \\ P(E') &= 1 - P(E) = 0.998 \end{aligned}$$

$$\begin{aligned} P(A) &= P(AB'E') + P(AB'E) + P(ABE') + P(ABE) \\ &= P(A | B'E').P(B'E') + P(A | B'E).P(B'E) + P(A | BE').P(BE') + P(A | BE).P(BE) \\ &= 0.001 \times 0.999 \times 0.998 \\ &\quad + 0.29 \times 0.999 \times 0.002 \\ &\quad + 0.95 \times 0.001 \times 0.998 \\ &\quad + 0.95 \times 0.001 \times 0.002 \\ &= 0.001 + 0.0006 + 0.0009 \\ &= 0.0025 \end{aligned}$$

$$\begin{aligned} P(J) &= P(JA) + P(JA') \\ &= P(J | A).P(A) + P(J | A').P(A') \\ &= 0.9 \times 0.0025 + 0.05 \times (1 - 0.0025) \\ &= 0.052125 \end{aligned}$$

$$\begin{aligned} P(AB) &= P(ABE) + P(ABE') \\ &= 0.95 \times 0.001 \times 0.002 + 0.95 \times 0.001 \times 0.998 \\ &= 0.00095 \end{aligned}$$

$$\begin{aligned} P(A'B) &= P(A'BE) + P(A'BE') \\ &= P(A' | BE).P(BE) + P(A' | BE').P(BE') \\ &= (1 - 0.95) \times 0.001 \times 0.002 + (1 - 0.95) \times 0.001 \times 0.998 \\ &= 0.00005 \end{aligned}$$

$$\begin{aligned} P(AE) &= P(AEB) + P(AEB') \\ &= 0.95 \times 0.001 \times 0.002 + 0.29 \times 0.999 \times 0.002 \\ &= 0.00058 \end{aligned}$$

$$\begin{aligned} P(AE') &= P(AE'B) + P(AE'B') \\ &= 0.95 \times 0.001 \times 0.998 + 0.001 \times 0.999 \times 0.998 \\ &= 0.001945 \end{aligned}$$

$$\begin{aligned} P(A'E) &= P(A'E'B) + P(A'E'B') \\ &= P(A' | BE').P(BE') + P(A' | B'E').P(B'E') \\ &= (1 - 0.95) \times 0.001 \times 0.998 + (1 - 0.001) \times 0.999 \times 0.998 \\ &= 0.996 \end{aligned}$$

$$\begin{aligned} P(JB) &= P(JBA) + P(JBA') \\ &= P(J | AB).P(AB) + P(J | A'B).P(A'B) \\ &= P(J | A).P(AB) + P(J | A').P(A'B) \\ &= 0.9 \times 0.00095 + 0.05 \times 0.00005 \\ &= 0.00086 \end{aligned}$$

$$P(J | B) = P(JB) / P(B) = 0.00086 / 0.001 = 0.86$$

$$\begin{aligned} P(MB) &= P(MBA) + P(MBA') \\ &= P(M | AB).P(AB) + P(M | A'B).P(A'B) \\ &= P(M | A).P(AB) + P(M | A').P(A'B) \\ &= 0.7 \times 0.00095 + 0.01 \times 0.00005 \\ &= 0.00067 \end{aligned}$$

$$P(M | B) = P(MB) / P(B) = 0.00067 / 0.001 = 0.67$$

$$P(B | J) = P(JB) / P(J) = 0.00086 / 0.052125 = 0.016$$

$$P(B | A) = P(AB) / P(A) = 0.00095 / 0.0025 = 0.38$$

$$\begin{aligned} P(B | AE) &= P(ABE) / P(AE) = [P(A | BE).P(BE)] / P(AE) \\ &= [0.95 \times 0.001 \times 0.002] / 0.00058 \\ &= 0.003 \end{aligned}$$

$$\begin{aligned} P(AJE') &= P(J | AE').P(AE') \\ &= P(J | A).P(AE') \\ &= 0.9 \times 0.001945 \\ &= 0.00175 \end{aligned}$$

$$\begin{aligned} P(A'JE') &= P(J | A'E').P(A'E') \\ &= P(J | A').P(A'E') \\ &= 0.05 \times 0.996 \\ &= 0.0498 \end{aligned}$$

$$P(JE') = P(AJE') + P(A'JE') = 0.00175 + 0.0498 = 0.05155$$

$$P(A | JE') = P(AJE') / P(JE') = 0.00175 / 0.05155 = 0.03$$

$$\begin{aligned} P(BJE') &= P(BJE'A) + P(BJE'A') \\ &= P(J | ABE').P(ABE') + P(J | A'BE').P(A'BE') \\ &= P(J | A).P(ABE') + P(J | A').P(A'BE') \\ &= 0.9 \times 0.95 \times 0.001 \times 0.998 + 0.05 \times (1 - 0.95) \times 0.001 \times 0.998 \\ &= 0.000856 \end{aligned}$$

$$P(B | JE') = P(BJE') / P(JE') = 0.000856 / 0.05155 = 0.017$$