INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ……. *FN / AN* Time: *2 hrs* Full marks: 40 No. of students: 90

Autumn 2013, Mid Semester Exams Dept: *Computer Sc & Engg.* Sub No: CS60047

B.Tech / M.Tech (Elective) Sub Name: ***Advanced Graph Theory***

Instructions: Answer all questions. Answer all parts of a question in the same place
Answers written using illegible handwriting and/or illegible logic will not be graded.

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1. [Counting Problems:] Answer the following with ***brief*** justification.
   a) What is the maximum number of edges in a graph with 10 vertices and 3 components?
   b) What is the number of tournaments with vertex set [5] (that is, 5 labelled vertices)?
   c) How many trees with vertex set [5] are possible with exactly 4 leaves?
   d) How many perfect matchings are possible in K_8, the complete graph of 8 vertices?
   e) What is the size of a minimum vertex cover of Q_6, the 6-dimensional hypercube?

   [3 X 5 = 15 marks]

2. The minimum cost cover is shown (in the boxes) along with the weight matrix of a complete bipartite graph $K_{5,5}$. Find the maximum weight matching from this information. Note that you have to identify the edges that belong to the maximum weight matching and also report the weight of the matching. Show the steps neatly with crisp justification (no verbose explanations please).

	0	**0**	**2**	**2**	**1**
4	4	1	6	2	3
5	5	0	3	7	6
7	2	3	4	5	8
4	3	4	6	3	4
6	4	6	5	8	6

[5 marks]

3. The preferences of a set of 3 men, { u, v, w }, and a set of 3 women, { a, b, c }, for their respective partners are given below. Based on this information, determine whether there are unstable pairs in the matching { (u, a), (v, b), (w, c) }. If so, find all unstable pairs and justify.

Preferences of the Men	Preferences of the Women
u: b > a > c	a: v > w > u
v: b > a > c	b: w > u > v
w: a > c > b	c: w > v > u

[5 marks]

4. Use Havel and Hakimi's recursive test to determine whether (5, 5, 4, 3, 2, 2, 2, 1) is a graphic sequence. If so, then re-construct the graph from the recurrence such that $d(v_1) = 5$, $d(v_2) = 5$, $d(v_3) = 4$, $d(v_4) = 3$, $d(v_5) = 2$, $d(v_6) = 2$, $d(v_7) = 2$, $d(v_8) = 1$. Your graph must show these vertex labels.

[5 marks]

5. The set of 3 length sub-sequences of the DNA sequence, ATTGCA, is { ATT, TTG, TGC, GCA }. The DNA sequencing problem starts with all possible sub-sequences of a given length from a DNA sequence and attempts to reconstruct the DNA sequence. You are given the following set of all sub-sequences of length 3 each obtained from a DNA sequence, S, of length 10. It is known that S starts with ATT:

   Sub-sequences of S = { ATG, ATT, GTT, TAT, TGC, TGT, TTA, TTG }

   Demonstrate how you will try to reconstruct S from this information. Note that decomposing a sequence into sub-sequences may lead to loss of information and hence your reconstruction may produce multiple candidates for S. If this happens in this example, then find all possible candidates. Working with an appropriate graph representation of the problem is desirable.

[5 marks]

6. Let M be a matching in a graph G, and let *u* be an M-unsaturated vertex. Prove that if G has no M-augmenting path that starts at *u*, then *u* is unsaturated in some maximum matching in G.
   [Hint: Consider the symmetric difference between M and a maximum matching M' that saturates *u*.] *Your proof must be concise and precise.*

[5 marks]