INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: …….*FN / AN* Time: *2 hrs* Full marks: 60 No. of students: 58

Autumn Mid Semester, 2007 Dept: *Comp. Sc & Engg.* Sub No: CS40005 / CS60047

B.Tech / M.Tech (Elective) Sub Name: ***Graph Theory***

Instructions: *Answer Questions 1 and 2, and any three from the rest.*
Answer all parts of a question in the same place

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1. Count the following. In each case, prove your answer in brief.
   (a) The number of perfect matchings in K_{2n}
   (b) The number of edges in the k-dimensional hypercube, Q_k
   (c) The number of trees with vertex set [n] having $n-2$ leaves

   [5 X 3 = 15 marks]

2. Prove (with a brief proof) or Disprove (with a counterexample) each of the following statements.
   (a) In every simple graph, the size of a maximum matching equals the size of a minimum vertex cover.
   (b) Every Eulerian simple graph with an even number of vertices has an even number of edges.
   (c) If the preference list of all men are identical, then every perfect matching is stable.

   [5 X 3 = 15 marks]

3. Let d_1 ,..., d_n be positive integers with $n \geq 2$. Prove that there exists a tree with vertex degrees d_1,…, d_n, if and only if $\sum d_i = 2n - 2$ [10 marks]

4. Let M be a matching in a graph G, and let *u* be an *M*-unsaturated vertex. Prove that if G has no *M*-augmenting path that starts at *u,* then *u* is unsaturated in some maximum matching M′ in G.
   [Hint: *Create M′ by examining* $M \Delta M_1$ *for some maximum matching* M_1] [10 marks]

5. Given a minimum weighted cover (*u, v*), how will you find a maximum weighted matching in a bipartite graph? [10 marks]

6. Explain the Hungarian Algorithm for finding a maximum weighted matching in a bipartite graph. [10 marks]