INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date:FN / ANTime: 2 hrsFull marks: 60No. of students: 48Autumn Mid Semester, 2006Dept: Comp. Sc & Engg.Sub No: CS40005 / CS60047B.Tech / M.Tech (Elective)Sub Name: Graph Theory

Instructions: Answer all questions. Answer all parts of a question in the same place

- 1. Indicate whether the following statements are True or False. For each True statement, give a brief proof. For each False statement, give a small counter-example.
 - (a) Every maximal trail in an even graph is an Eulerian circuit.
 - (b) For $k \ge 2$, a k-regular bipartite graph has no cut edge.
 - (c) If the preference lists of all women are identical, then there exists only one stable matching.[4 X 3 = 12 marks]
- 2. Count the following. In each case, prove your answer.
 - (a) The number of spanning trees of K_n
 - (b) The number of edges in the k-dimensional hypercube, Q_k
 - (c) The number of directed graphs with vertices, $v_1, ..., v_n$.
 - (d) The number of perfect matchings in K_{2n}
 - (e) The number of automorphisms of $K_{n,n}$ [5 X 5 = 25 marks]
- T is a given minimum cost spanning tree of a given graph, G. Give an algorithm to modify T to get a minimum cost spanning tree of G e, where e is any given edge of G. Assume that e is not a cut edge. Prove the correctness of your algorithm. [5 marks]
- 4. For every graph, G, prove that $\beta(G) \le 2\alpha'(G)$. For a given k, construct a simple graph G with $\alpha'(G) = k$ and $\beta(G) = 2k$. [4+4=8 marks]
- 5. You are given a minimum edge cover of a bipartite graph, G.
 - (a) Give an algorithm for finding a maximum matching in G from the minimum edge cover
 - (b) Give an algorithm for finding a minimum vertex cover of G from the maximum matching found in part (a).[5+5=10 marks]