INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ……….*FN / AN* Time: *2 hrs* Full marks: 30 No. of students:

Autumn Mid Semester, 2005 Dept: *Computer Science & Engg.* Sub No: CS40005

B.Tech (Elective) Sub Name: ***Applied Graph Theory***

Instructions: *Answer all questions. Answer all parts of a question in the same place*

---

1. **Tournaments:** [2 + 4 = 6 marks]
   (a) What is a tournament? What is a king of a tournament?
   (b) Prove that if a tournament has no vertex with zero in-degree, then it has at least 3 kings.

2. **Graphic Sequences:** [2 + 4 = 6 marks]
   (a) What is a graphic sequence?
   (b) Determine whether [5, 5, 5, 4, 3, 2] is a graphic sequence. Show the steps of your algorithm. If it is indeed a graphic sequence, then produce a graph as a witness.

3. **Euler graphs:** [3 + 3 = 6 marks]
   Give a formal proof or a counter-example for the following statements:
   (a) The degree sum of every Eulerian bipartite graph is divisible by 4.
   (b) Every simple Eulerian graph with an even number of vertices has an even number of edges.

4. **Trees and Counting:** [2 + 4 = 6 marks]
   (a) Count the number of spanning trees of K_n and express it as a function of *n*.
   (b) Let G be a graph obtained from K_n by deleting an edge. Count the number of spanning trees of G and express it as a function of *n.* Give the argument for counting.

5. **Stable matchings:** [2 + 4 = 6 marks]
   (a) Give the formal definition of a stable matching.
   (b) Consider the following algorithm for finding a matching between men and women. We ask each person to rank each member of the opposite sex in order of his / her preference. Let *rank(x, y)* denote the rank of *y* in the preference list of *x*. The weight of a match between man *x* and woman *y* is the sum of *rank(x, y)* and *rank(y, x)*. We find the maximum weighted matching using the Hungarian algorithm. Does this algorithm give us a stable matching? Give a formal proof (if yes) or a counter-example (if no).