INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ..... *FN/AN* Time: *2 Hrs* Full Marks: *60* No. of students: 38
Autumn Mid Semester Exams, 2004 Dept: *Computer Sc. & Engg.* Sub No: *CS40005*
B.Tech (Elective) Sub Name: *Applied Graph Theory*

Instructions: *Answer all questions*
All parts of a question must be answered in the same place

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1. (a) Give an algorithm that computes the degree sequence of a tree from its Prufer sequence *without constructing the tree*. Justify the correctness of your algorithm.

   (b) Given a sequence of n integers, give a method to determine whether it can be the degree sequence of a tree. Demonstrate your method on the sequence $[3, 3, 2, 1, 1]$.

   (c) Given the degree sequence of a tree of n vertices, use the Prufer coding scheme to count the number of trees having vertex set $[n]$ and the same degree sequence. Demonstrate your method on the sequence $[4, 4, 3, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1]$.

   [10+10+10 = 30 marks]

2. One chapter of Graph Theory has k exercise problems. The Graph Theory class has n students. Each student submits a list of problems that he/she would like to attempt. The teacher wants to minimize the total number of solution submissions that he/she has to check, subject to the following contraints:

   - Each student attempts one or more problems from his/her list, and
   - Each problem is attempted by one or more students

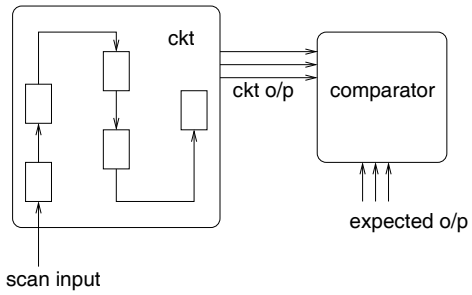
   (a) Extend the bipartite matching algorithm to distribute the problems among the students in a way that minimizes the total number of solutions that the teacher has to check. Prove the correctness of your algorithm.

   (b) Let us now consider an extension of the above problem, where each student orders all problems in order of preference and submits it to the teacher. The teacher also associates with each problem, i, an upper bound k_i on the number of students who can be given that problem, such that $\sum_i k_i = n$. Each student will therefore get only one problem. An allocation is *stable* if no two students can get preferable problems by exchanging the ones assigned to them. Formulate this as a weighted bipartite maximum matching problem, and prove that the assignment is stable.

   [10+10 = 20 marks]

3. A sequential circuit consists of one or more flops (memory elements) and combinational logic. The *state* of the circuit is given by a bit-vector consisting of the current logic values in the flops of the circuit. In order to test such circuit chips, the flops are connected through a *scan chain*, so that the set of flops can be viewed as a shift-register. One end of the scan chain is an input pin of the chip. We can bring the circuit to a given state, by serially inserting the bit-vector for the

desired state through this pin and shifting it into the flops of the circuit. If there are k flops, then inserting a completely new k-bit vector requires k shifts. The k-bit vector is called a test pattern. We wish to successively test the circuit for all k-bit test patterns, and compare the output with the expected output.



We can optimize the number of shifts required by appropriately ordering the test patterns. For example, if the first m bits of a bit-vector is the same as the last m-bits of the previous bit-vector, then we can insert the new bit-vector by only inserting the last $k-m$ bits of the new vector which requires only $k-m$ shifts.

Reduce the problem to a known graph theoretic problem. Give an algorithm that produces an ordering among the test patterns so that a minimum number of shifts are required to test the circuit for all k-bit test patterns. Prove that your algorithm produces the best possible ordering.

[10 marks]
