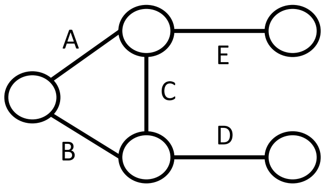
INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ………… *FN / AN* Time: *3 hrs* Full marks: 100 No. of students: 92

Autumn 2013, End Semester Exams Dept: *Computer Sc & Engg.* Sub No: CS60047

B.Tech / M.Tech (Elective) Sub Name: ***Advanced Graph Theory***

Instructions: Answer all parts of a question in the same place
Answers written using illegible handwriting and/or illegible logic will not be graded.



<u>PART-1</u>: *Answer all questions in this part*

1. Answer the following questions with a brief justification
   a) Draw the dual graph of the graph shown
   b) Draw the line graph of the graph shown
   c) Suppose T is a tree and $\chi(T;3) = 48$. How many vertices does T have?
   d) A connected plane graph, G, has 10 vertices and 15 edges. How many faces do G*, the dual graph of G, have?
   e) A graph G with *n* vertices and *e* edges has a maximum matching of size k (that is, $\alpha'(G) = k$). Then what is the size of the minimum vertex cover of the line graph, L(G), of G?

[3 X 5 = 15 marks]

2. Indicate whether the following statements are True / False. No justification is required.
   (a) If G is non-planar, then $\chi(G) > 4$
   (b) A planar graph has a cut vertex iff its dual has a cut vertex.
   (c) The complement of the 3-dimensional hypercube Q_3 is non-planar.
   (d) An undirected graph G has a strongly connected orientation only if G is 2-connected
   (e) The complement of a color critical graph cannot be color critical.
   (f) G is isomorphic to its line graph L(G) iff G is 2-regular.
   (g) A graph which is both Hamiltonian and Eulerian and has a chromatic number 2 must be planar.
   (h) 15 people cannot shake hands in a way that each person shakes hand with 5 distinct people.
   (i) Finding the minimum vertex cover of a bipartite graph is a NP-hard problem
   (j) Two non-isomorphic graphs can have the same line graph.

[1 X 10 = 10 marks]

3. Consider the graphs $G_1 = \overline{C_6}$, $G_2 = \overline{C_3 + C_3}$, $G_3 = K_3 \vee K_3$, $G_4 = Q_3$.
   a) Which of these graphs are planar? Give a planar drawing for the planar one and a brief justification for the rest.
   b) Write down the chromatic number for each of these graphs.
   c) Write down the vertex connectivity for each of these graphs.

[7 + 4 + 4 = 15 marks]

<u>PART-2</u>: *Answer any three from this part*

4. [Planarity:]

   a) State and prove Euler's formula for plane graphs.

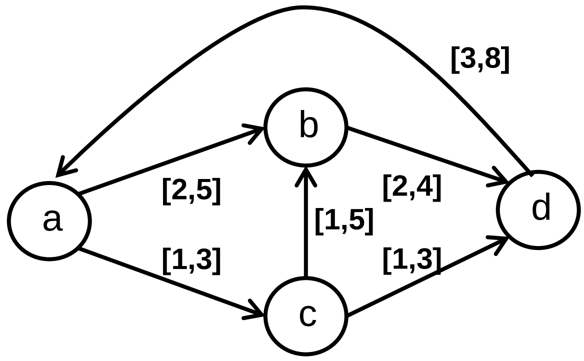
   b) Indicate whether Euler's formula holds for these plane graphs (with justification):

      (i) graphs having multiple components (ii) graphs which are not simple

   c) Use Euler's formula to count the number of edges in every 10-vertex plane graph which is isomorphic to its dual.

[8 + 8 + 4 = 20 marks]

5. [Network Flows]



   a) In the above network, the vertices represent the cities in a country. An airlines company wishes to operate flights along the edges of this network (in the indicated directions). For each connection, there is a minimum number of flights that must be operated and a maximum number of flights that can be operated. The company wishes to determine whether a set of aircrafts can be circulated along this network so that all connections are satisfied with respect to their lower and upper bounds. Note that circulation means that no aircraft is stranded at a city. Using this network, demonstrate how this circulation problem can be transformed into a max-flow problem. Your answer must show the transformed network after each step of transformation and also how a feasible flow may be found after solving the max-flow problem.

   b) Consider a round-robin chess tournament with n players with each player playing every other player exactly once. A win scores 1 point for the winner and 0 points for the loser, while a draw scores ½ points for each player. We are given a set of final scores (S_1, …, S_n) for the players with $0 \leq S_i \leq n - 1$. We want to check whether these scores are feasible (for example, in a three-player tournament, a set of final scores of (2, ½, 2) is impossible. Model this problem as a network flow problem. Draw the network you will use to test for the feasibility of the scores (3, 2½, ½, 0) for a tournament with 4 players.

[10 + 10 = 20 marks]

6. [Graph Coloring]
   a) Explain Mycielski's construction.
   b) How shall we order the vertices of an interval graph so that the greedy colouring algorithm yields a colouring with minimum number of colours? What is the justification behind choosing this ordering?
   c) Indian Railways is planning to run a new train from Howrah to Chennai. It wants to add reservation quotas between select cities. The number of berths needed in the quota from one station to another is shown in the table. Each coach of the train has 60 berths. The Railways wants to determine the minimum number of coaches needed to accommodate the quotas. Formulate this as a graph colouring problem. Clearly indicate the set of vertices and what they represent. What do the edges represent? Is this a special type of graph colouring problem? If so, can you compute the number of coaches directly from the following table?

	KGP	BBS	VZM	VSKP	BZA	MAS
HWH	10	45	70	--	--	200
KGP		20	--	--	--	50
BBS			50	40	--	50
VZM				30	30	50
VSKP					40	40
BZA						20

Assume that the stations appear in this sequence in the route.

[4 + 6 + 10 = 20 marks]

7. [Miscellaneous]
   a) Define the following and give an example for each:
      (I) Eulerian graph (II) Hamiltonian closure of a graph
      (III) Kuratowski subgraph (IV) Chromatic recurrence
   b) State the following theorems (only statements are needed – no proofs):
      (A) Menger's Theorem (Cuts and Connectivity)
      (B) Brook's Theorem (Graph Colouring)
      (C) König and Egerváry Theorem (Matchings)
   c) Prove that the degree sum of every Eulerian bipartite graph is divisible by 4.

[8 + 6 + 6 = 20 marks]