INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ……….*FN / AN*   Time: *3 hrs*   Full marks: 100   No. of students: 90

Autumn End Semester, 2009   Dept: *Comp. Sc & Engg.*   Sub No: CS40005 / CS60047

B.Tech / M.Tech (Elective)   Sub Name: ***Graph Theory***

Instructions: ***All questions in Part-1 must be answered on the Question paper.***
Answer any three from the questions in Part-2.

Part-1: Compulsory (3-questions)– Answer all questions in this part. Write in the boxes.

Roll Number: [ ]   **Name:** [ ]

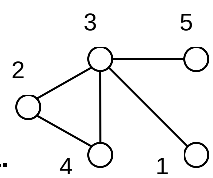

Fig 1.

1. Answer the following questions (no justifications are required):   [10 X 2 = 20 marks]

   (a) What is the maximum number of kings in a tournament with n vertices?

   Ans:

   (b) What is the number of faces in a plane graph with *10* vertices and *20* edges?

   Ans:

   (c) Count the number of spanning trees of the graph shown in Fig 1.

   Ans:

   (d) Count the number of ways in which the graph shown in Fig 1 can be colored with 6 colors. Hint: *Use chromatic recurrence.*

   Ans:

   (e) Suppose M_1 and M_2 are 1-factors of K_n and let $H = M_1 \Delta M_2$ be the symmetric difference between M_1 and M_2. What are the values of $\Delta(H)$ if (i) $M_1 \neq M_2$ and (ii) $M_1 = M_2$?

   Ans:

   (f) What is the maximum number of edges in a 10 vertex 3-colorable graph?

   Ans:

(g) Suppose T is a tree and $\chi(T;3) = 24$. How many vertices does T have?

Ans:

(h) Count the number of walks of length 10 in K_5. Assume that vertices are labeled.

Ans:

(i) Determine all *r,s* such that $K_{r,s}$ is planar

Ans:

(j) $\chi(G_1) = k_1$ and $\chi(G_2) = k_2$. What are the values of $\chi(G_1 + G_2)$ and $\chi(G_1 \vee G_2)$?

Ans:

2. State the following: [3 X 5 = 15 marks]

(a) Brook's Theorem:

(b) Definition of a DeBruijn Cycle:

(c) Kuratowski's Theorem:

(d) Definition of Line Graph:

(e) Max-Flow Min-Cut Theorem:

3. Indicate whether the following statements are True/ False. No justification is required.

[2 X 10 = 20 marks]

(a) If a tournament has a single king, then it is not strongly connected.

(b) If the dual of a plane graph G is Eulerian, then G is 2-colorable.

(c) The transportation problem is NP-hard.

(d) 15 people cannot shake hands in a way that each person shakes hand with 3 distinct people.

(e) If $\delta(G) < k—1$, then G cannot be k-critical

(f) Eulerian graphs are always 2-edge connected

(g) Two non-isomorphic graphs can have the same line graph

(h) Two non-isomorphic connected plane graphs can have the same dual graph

(i) Minimum edge covers of graphs are cycle free

(j) Every tree of even order has a perfect matching

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Part-2: Answer any three. All parts of a Question must be answered in the same place

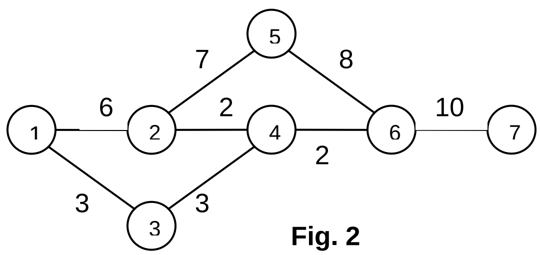

Fig. 2

4. [Flows] [7 + 8 = 15 marks]

(a) Use the augmenting path algorithm to establish a maximum flow from vertex-1 to vertex-7 in the graph shown in Fig 2. Show the residual network at the end of each augmentation and specify the minimum cut that the algorithm obtains on termination.

(b) Suppose the graph of Fig 2 represents a computer network. Nodes 1 and 4 are client nodes which generate database queries and nodes 5 and 7 are server nodes which are capable of executing the queries. The other nodes can simply forward the queries. The propagation of each query generates one unit of network traffic. The capacity of an edge represents an upper-bound on the network traffic through that link. The rates at which queries are generated at Node-1 and Node-4 are 6 and 4 respectively, and the rates at which queries are executed in Node-5 and Node-7 are 15 and 10 respectively. We wish to determine whether the queries can be forwarded from the client nodes to the server nodes while respecting the link capacities.

(i) Model this as a transportation problem, indicating the supplies and demands.

(ii) Convert this problem into a max-flow problem. Draw the corresponding graph showing the source, the sink, and capacities of all edges. Indicate how the solution of the max-flow problem will help you in deciding the answer to the original problem.

5. [Planarity] [7+2+6= 15 marks]

(a) State and prove Euler's formula for plane graphs.

(b) Draw a 4-vertex plane graph which is isomorphic to its dual.

(c) Use Euler's formula to prove that every n-vertex plane graph isomorphic to its dual has $2n - 2$ edges.

6. [Colorings] [4 + 5 + 6 = 15 marks]

(a) Use Mycielski's construction on K_3 and draw the resulting graph as a plane graph.

(b) Define Chromatic Recurrence. Use chromatic recurrence to compute $\chi(C_4; k)$.

(c) Consider the flight map of an airlines company with n cities, and k flights. Each flight connects exactly two cities and is shown in the map as a curve joining the two cities. We must color the flights (edges) incident on each city with different colors. Reduce this problem into a vertex coloring problem. Show your reduction using the graph of Fig 2 (ignore the edge capacities in Fig 2 for this problem). [Hint: *Use Line Graphs*]

7. [Miscellaneous] [4 + 5 + 6 = 15 marks]

(a) Reconstruct the tree from the Prufer sequence [2, 3, 1, 1, 4]

(b) Give an example to show that the augmenting path algorithm for maximum bipartite matching is not appropriate for non-bipartite graphs. Which algorithm solves the problem?

(c) Give a necessary and sufficient condition for $d_1, \ldots, d_n$ to be the degree sequence of a tree. Provide a brief proof (of necessity and sufficiency) to justify your claim.