INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ……….*FN / AN* Time: *3 hrs* Full marks: 99 No. of students: 58

Autumn End Semester, 2007 Dept: *Comp. Sc & Engg.* Sub No: CS40005 / CS60047

B.Tech / M.Tech (Elective) Sub Name: ***Graph Theory***

Instructions: *Answer Question 1 and 2, and any three from the rest.*
Question 1 and 2 must be answered on the question paper itself.
Answer all parts of a question in the same place

Roll No: Name:

1. Enter the numerical answer to the following questions in the box provided. [8X3 = 24 marks]

   (a) In how many ways can we color C_4 properly with 5 colors?

   (b) What is the maximum *k* for which the *k*-dimensional hypercube, Q_k, is planar?

   (c) Among all graphs, G, which are *k*-chromatic and *k* > 4, what is the minimum possible value of $\omega(G)$? $\omega(G)$ denotes the size of the maximum clique in G.

   (d) How many edges does the line graph of K_4 have?

   (e) What is the connectivity of the Turan graph, $T_{10,3}$?

   (f) Let T be an *acyclic orientation* of K_n. How many kings can T have?

   (g) A bipartite graph of 5 vertices and 4 edges has an edge cover of size 3. What is the size of the maximum independent set in G?

   (h) What is the minimum number of edges among all non-planar graphs?

2. Indicate whether the following statements are True / False in the box provided with an accurate reason in the line provided. [10 X 3 = 30 marks]

   (a) Every 4-colorable graph is planar.

   (b) Every *k*-critical graph is k–1 edge connected.

(c) A proper coloring of G can never be a proper coloring of the compliment of G.

(d) A matching in G becomes an independent set in the line graph, L(G), of G.

(e) Every *k*-edge connected graph is *k*-connected.

(f) Every tree has at most one perfect matching

(g) Every closed even walk contains an even cycle.

(h) Every graph having a 2-factor also has a 1-factor.

(i) Deletion of an edge of a graph reduces its connectivity by at most 1.

(j) Every graph without a cut-vertex has a spanning cycle.

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3. [Planarity:]
   (a) State Euler's formula for plane graphs.
   (b) Indicate whether Euler's formula holds for these graphs (with justification):
   (i) graphs having multiple components (ii) graphs which are not simple
   (c) Use Euler's formula to count the number of edges in every 10-vertex plane graph which is isomorphic to its dual.

[2 + 6 + 7 = 15 marks]

4. [Graph coloring:]
   (a) State the Greedy coloring algorithm. Give an example to demonstrate that it does not produce an optimal ordering for every ordering of vertices.
   (b) Given an optimal coloring of a graph G, how would you produce an ordering of the vertices such that the greedy algorithm also produces an optimal coloring? Does it produce the same coloring?
   (c) Suppose G is 2k–1 regular and 2k-critical. Then how many vertices does G have?
   [Hint: *State and use Brook's Theorem*]

[5 + 5 + 5 = 15 marks]

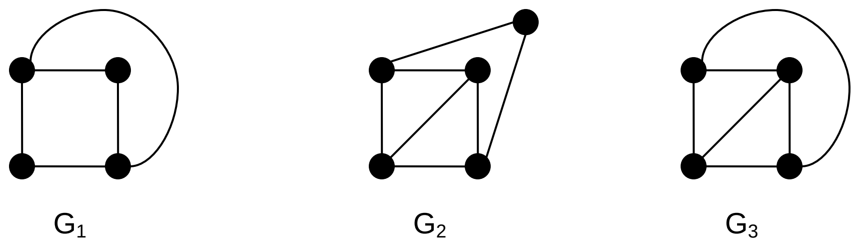
5. [Network Flows:]
   (a) Consider a variant of the max-flow problem where instead of finding the maximum total flow from the source node, *s*, to the sink node, *t*, we wish to find a single path from *s* to *t* with the highest capacity. Show that this problem reduces to a well known graph problem.
   (b) During the semester examinations, several subjects from each department are to be scheduled on the examination halls. In a given day, the k^{th} department wishes to schedule m_k subjects. Subjects from the same department must be scheduled in different rooms. The j^{th} hall can accommodate up to n_j subjects. Show how to use network flows to test whether the constraints can be satisfied.
   (c) Outline the steps in transforming a feasible flow problem into an ordinary maximum flow problem. Define all problems used in the intermediate steps of the transformation.

[5 + 5 + 5 = 15 marks]

6. [Connectivity:]

   (a) How do we construct the line graph, L(G), of a graph G?

   (b) Indicate which of the following graphs are line graphs of some graph (with brief justification):



   (c) State Menger's Theorem and use it to prove that: $\kappa'_G(x, y) = \lambda'_G(x, y)$

[2 + 6 + 7 = 15 marks]

7. [Miscellaneous:]

   (a) Let G be a self-complimentary graph. Prove that G has a cut-vertex if and only if G has a vertex of degree 1.

   (b) A batch of n students are asked to submit choices for k projects ($k > n$). Each project can be given to exactly one student. For each project, the choices of at most the top three students (among those who opted for that project) are recorded (for some projects fewer than three students may have opted). No student is allowed to leave until exactly three of his/her choices have been recorded. Given that each student has recorded exactly three options and for no project have we recorded more than three choices, is it guaranteed that we will be able to give each student a project from his/her recorded options?
   [Hint: *Check Hall's Condition*]

   (c) The following table illustrates a number of possible duties for the drivers of a bus company. We wish to ensure at the lowest possible cost, that at least one driver is on duty for each hour of the planning period (9 AM to 5 PM). Formulate and solve this scheduling problem as a shortest path problem. Draw the graph on which you will solve the shortest path problem.

Duty hours	9 – 1	9 – 11	12 – 3	12 – 5	2 – 5	1 – 4	4 – 5
Cost	30	18	21	38	20	34	9

[5 + 5 + 5 = 15 marks]