INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ……….*FN / AN* Time: *3 hrs* Full marks: 80 No. of students: 48

Autumn End Semester, 2006 Dept: *Comp. Sc & Engg.* Sub No: CS40005 / CS60047

B.Tech / M.Tech (Elective) Sub Name: ***Graph Theory***

Instructions: ***All questions in Part-1 must be answered on the Question paper.***
Answer any three from the questions in Part-2.

Part-1: Compulsory (3-questions)– Answer all questions in this part. Write in the boxes.

Roll Number: [ ] **Name:** [ ]

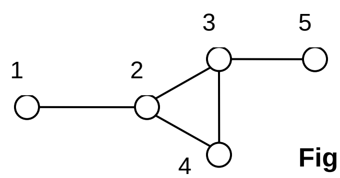

Fig 1.

1. Answer the following questions (no justifications are required): [10 X 2 = 20 marks]

(a) G is a tournament with *n* vertices and *x* is a king of G. What is the maximum possible distance from *x* to other vertices of G? Ans:

(b) What is the minimum number of components for graphs having *n* vertices and *k* edges?
Ans:

(c) Count the number of spanning trees of the graph shown in Fig 1.
Ans:

(d) Count the number of ways in which the graph shown in Fig 1 can be colored with 6 colors. Hint: *Use chromatic recurrence.*
Ans:

(e) Suppose M_1 and M_2 are 1-factors of K_n and let $H = M_1 \Delta M_2$ be the symmetric difference between M_1 and M_2. If $M_1 \neq M_2$ then what is the value of $\Delta(H)$?
Ans:

(f) G and H are simple graphs having two components each. What is the maximum number of components in the complement of G+H?
Ans:

(g) Suppose T is a tree and $\chi(T;3) = 24$. How many vertices does T have?

Ans:

(h) For what values of *n* is the compliment of $K_n + K_n$ planar?

Ans:

(i) A graph G having *n* vertices and *e* edges has a maximum matching of size k (that is, $\alpha'(G) = k$). Then what is the size of the minimum vertex cover of the line graph, L(G), of G?

Ans:

(j) $\chi(G_1) = k_1$ and $\chi(G_2) = k_2$. What are the values of $\chi(G_1 + G_2)$ and $\chi(G_1 \vee G_2)$?

Ans:

2. State the following: [1 X 5 = 5 marks]

(a) Kuratowski's Theorem:

(b) Euler's Formula:

(c) Cayley's Formula:

(d) Four color theorem:

(e) Menger's Theorem:

3. Indicate whether the following statements are True/ False. No justification is required.

[1 X 10 = 10 marks]

(a) If G is non-planar, then $\chi(G) > 4$

(b) A planar graph has a cut vertex iff its dual has a cut vertex.

(c) The complement of the 3-dimensional hypercube Q_3 is non-planar.

(d) An undirected graph G has an strongly connected orientation only if G is 2-connected

(e) The complement of a color critical graph cannot be color critical.

(f) G is isomorphic to its line graph L(G) iff G is 2-regular.

(g) A graph which is both Hamiltonian and Eulerian and has a chromatic number 2 must be planar.

(h) 15 people cannot shake hands in a way that each person shakes hand with 5 distinct people.

(i) Finding the minimum vertex cover of a bipartite graph is a NP-hard problem

(j) Two non-isomorphic graphs can have the same line graph.

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Answer any three from the questions in Part-2.

Part-2: Answer any three. All parts of a Question must be answered in the same place

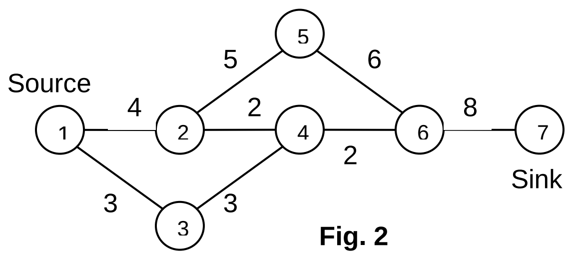

Fig. 2

4. [Flows] [8 + 7 = 15 marks]

(a) A flow x is *even* if for every edge (i,j), the flow x_{ij} through the edge is an even number; it is *odd* if for every edge (i,j), x_{ij} is an odd number. Either prove that each of the following claims are true or give a counterexample for them.

(i) If all edge capacities are even, then the graph has an even maximum flow

(ii) If all edge capacities are odd, then the graph has an odd maximum flow

(b) Use the augmenting path algorithm to establish a maximum flow from vertex-1 to vertex-7 in the graph shown in Fig 2. Show the residual network at the end of each augmentation and specify the minimum cut that the algorithm obtains on termination.

5. [Matching] [8 + 7 = 15 marks]

(a) Suppose X,Y are the partite sets of a bipartite graph G. Let m be the smallest degree in X and M be the largest degree in Y. Prove that if $m \geq M$, then G has a matching which saturates X. Hint: *Use Hall's condition.*

(b) Define the circulation problem. Formulate the bipartite prefect matching problem as a circulation problem. Hint: *Given a bipartite graph, G, show how we may construct another graph which has a feasible circulation iff G has a perfect matching.*

6. [Planarity] [9 + 6 = 15 marks]

(a) Show that any planar graph G with $\delta(G) \geq 5$ must have at least 12 vertices.

(b) Draw a *non-planar* graph G with at most 7 vertices that satisfies two properties, namely that G has a separating 2-set, S = {x, y}, and that each S-lobe of G is planar.

7. [Colorings] [9 + 6 = 15 marks]

(a) In the map coloring problem we are required to color the countries demarcated by borders so that neighboring countries (that is, ones sharing a common border) cannot have the same color.

(i) What is the worst case number of colors required to color a map?

(ii) In the map coloring problem we assumed that neighboring countries should share some positive length of border. Suppose we now call two countries to be neighbors even if their borders touch a single common point. Does this new definition affect the worst case number of colors required? Justify.

(iii) In the map coloring problem we assumed that a country consists of a single region. If we drop this restriction (thereby allowing a country to have many disjoint regions) and insist that all regions of a country must receive the same color, then does it affect the worst case number of colors required? Justify.

(b) Draw a small graph and produce an ordering of its vertices for which Greedy coloring fails to use the minimum number of colors.

8. [Paths and Connectivity] [10 + 5 = 15 marks]

(a) A farmer wishes to transport a truckload of eggs from one city to another city through a given road network. The truck will incur a certain amount of breakage on each road segment: let w_{ij} denote the fraction of the eggs broken if the truck traverses the road segment (i,j). How should the truck be routed to minimize the total breakage? Formulate this problem as an instance of the standard shortest path problem (No algorithm required). Hint: *Given an instance of the farmer's problem, show how we may create a graph such that the shortest path of the graph yields a solution to the farmer's problem. Clearly define the vertices, edges and edge costs of this graph*.

(b) What is a *common system of distinct representatives (CSDR)*? State a necessary and sufficient condition for two families of sets, **A** = {A_1, …, A_m} and **B** = {B_1, …, B_m}, to have a CSDR. (No proof is required)