INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: ……….*FN / AN* Time: *3 hrs* Full marks: 50 No. of students: 12

Autumn End Semester, 2005 Dept: *Computer Science & Engg.* Sub No: CS40005

B.Tech (Elective) Sub Name: ***Applied Graph Theory***

Instructions: *Answer any five questions. Answer all parts of a question in the same place*

1.

(a) A connected plane graph G has 10 vertices and 20 edges.

i. Compute the number of faces in the dual graph G* of G.
ii. Show that we need at least three colors to color G.

(b) State and prove the 5-color theorem.

[5 + 5 = 10 marks]

2.

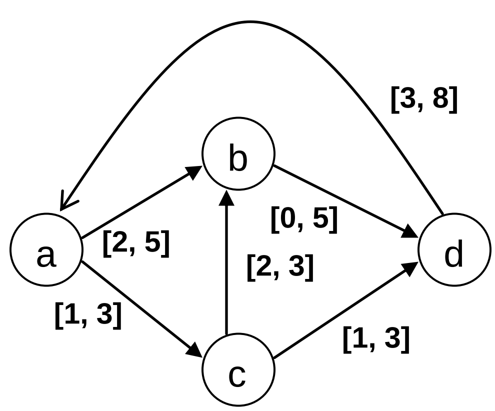
(a) All departments send their faculty members for invigilation duties during the semester examinations. The i^{th} department sends m_i invigilators. The examination section has to schedule these candidates into k examination halls over 5 days, such that:

- Each hall j gets a given number n_j of invigilators during each examination.
- All candidates may not be assigned a duty, but no candidate should be assigned more than one duty.
- The team of invigilators at a given hall in a given date must be constituted from members of different departments.

Formulate this problem as a network flow problem. You should clearly state what the edges and vertices of your network represent. You should also state how the formulation may be used to test whether a solution exists.

(b) Show the steps in converting a feasible circulation problem into a maximum flow problem. Demonstrate the steps on the following instance of a feasible circulation problem.

[5 + 5 = 10 marks]



3.

(a) Prove that if G is a color critical graph, then the graph G* generated from it by applying Mycielski's construction is also color critical.

(b) Indian Railways is planning to run a new train through k stations, $C_1, \ldots, C_k$ (in this succession). It wants to add reservation quotas between select cities. The number of berths needed in the quota from C_i to C_j is denoted by Q_{ij}. Each coach of the train has 30 berths. The Railways wants to determine the minimum number of coaches needed to accommodate the quotas. Reduce the problem to a graph coloring problem and give an efficient algorithm for finding an optimal solution.

[5 + 5 = 10 marks]

4.

(a) State Menger's theorem, and use it to prove the following statement. If x and y are distinct vertices of a graph G, then the minimum size of an x,y-disconnecting set of edges equals the maximum number of pair-wise edge disjoint x,y-paths.

(b) Let $A = (A_1, \ldots, A_m)$ be a collection of subsets of a set Y. A "*system of distinct representatives*" (SDR) for A is a set of distinct elements, $a_1, \ldots, a_m$, in Y such that $a_i \in A_i$. Using Hall's Theorem, prove that A has an SDR iff $| \bigcup_{i \in S} A_i | \geq |S|$ for every $S \subseteq \{1, \ldots, m\}$.

[5 + 5 = 10 marks]

5.

(a) Suppose a graph G has 9 vertices, each of degree 5 or 6. Prove that at least 5 vertices have degree 6 or at least 6 vertices have degree 5.

(b) A digital sequential circuit can be represented by a finite state transition system. Suppose we are given the power consumption for each state transition – let p_{ij} denote the power consumed in the transition from state s_i to state s_j. The peak power on a path from state s_i to state s_j is the maximum among the power consumed in the transitions on the path. We want to find the path having minimum peak power among all paths from s_i to s_j. Reduce this problem to a known graph problem and sketch an efficient algorithm.

[5 + 5 = 10 marks]

6. Give examples to demonstrate the working of the following algorithms.

(a) Hungarian algorithm
(b) Pre-flow push algorithm
(c) Edmond's blossom algorithm

[4 + 3 + 3 = 10 marks]