Foundation of Computer Science (CS60001)

Solution-11

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1 Solution

1. A graph G on n vertices is an n-cycle if and only if G is connected with each vertex having degree 2. Connectedness of a graph can be checked in polynomial time. Also, it is straightforward to check whether each vertex in a graph has degree 2. It follows that IS-HAM-CYCLE is in P and so cannot be NP-Complete unless P = NP.

2. From R and S, their NFA deciders N_R and N_S, can be constructed in polynomial time (in fact, linear time). However, if we construct the corresponsing DFAs D_R and D_S completely and then show them to be equivalent by constructing their parallel composition, then D_R and D_F will need exponential space. We notice that if Q is the set of states of the NFA, then the corresponding DFA will have no more than 2|Q| states. So in the following nondeterministic decider of EQ_{REX}, we club the two steps of conversion from the NFAs to the DFAs with equivalence checking together. The decider of EQ_{REX} = Input w = ⟨R, S⟩:
   (a) construct NFAs N_R and N_S from R and S, respectively;
   (b) let PS (present state pair) be $\langle\ q_R^0,\ q_s^0\ \rangle$, comprising the initial states of N_R and N_S;
   (c) repeat the followig steps $2^{|Q1|+|Q2|}$ times:

(d) choose nondeterministically one of $2^{|Q_1|}$ subsets of states for the first member of NS, the next state pair, and one of $2^{|Q2|}$ subsets for the second member;
(Ths step needs $|Q1| + |Q2|$ space) (e) check if the members of the NS-pair is reachable from the PS-pair on any input symbol; if not, reject;
(f) if one member of NS contains an accept state and the other does not, reject; else, if both the members of the NS-pair are accept states, accept; if neither of the members of NS contains an accept state, then PS $\leftarrow$ NS;
(g) reject; (since none of the above accepts)