

Coverage maximization under resource constraints using proliferating random walks

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Abstract. Dissemination of information has been one of the prime needs in almost every kind of communication network. The existing algorithms for this service, try to maximize the coverage, i.e., the number of distinct nodes to which a given piece of information could be conveyed under the constraints of time and energy. However, the problem becomes challenging for unstructured and decentralized environments. Due to its simplicity and adaptability, random walk (RW) has been a very useful tool for such environments. Different variants of this technique have been studied. In this paper, we study a history-based non-uniform proliferating random strategy where new walkers are dynamically introduced in the sparse regions of the network. Apart from this, we also study the breadth-first characteristics of the random walk-based algorithms through an appropriately designed metrics.

Keywords. Coverage; resource constraints; random walk; proliferation; sparsity.

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1. Introduction

Search and dissemination are the key requirements in any kind of communication networks such as peer-to-peer networks [1,2], delay tolerant networks [3], mobile social networks [4], internet of things [5], sensor networks [6], etc. Usually, the process has to be initiated from a single node and due to lack of centralized control and the spatial nature of many such networks, the process needs to travel through the neighbours of the nodes. However, in most of the cases due to the *ad-hoc* nature of the structure, an individual node does not have any information about the network except about its immediate neighbours. Moreover, the available energy in a node is also limited (e.g., nodes in wireless sensor networks [7], internet of things [8], mobile *ad-hoc* networks [9,10], etc.). We

make the simplifying assumption that each passing message takes equal energy and there is a constraint on the total amount of energy used for passing messages.

Flooding and the single random walk (1-RW) are the two fundamental algorithms in this direction. In flooding, at each time step a node that has the copy of the message forwards the message to each of its neighbours except from which it has received the message. In RW-based algorithms, a single message packet is modelled as a random walker. At each time step, there is a single copy of the message which is forwarded to a randomly selected node of the current neighbour. The existing works in the direction of achieving better coverage are either based on flooding or on single random walk or their combination. In this work we focus on a special class of proliferating RW-based algorithm as described in the following.

A class of algorithms based on proliferating random walk, where the number of walkers gradually increases with time following some rule was introduced in [11]. Later, a time-varying proliferation was proposed in [12] where the per walker per time step proliferation rate was appropriately varied to achieve an optimal use of time and energy. As an immediate next step, the notion for producing optimal coverage under a stricter time constraint was proposed in [13]. The main idea in this work was to dynamically sense the concentration of the walkers and proliferate the walkers only if the estimated density is small enough. In order to sense the concentration of the walkers dynamically and in a decentralized fashion, Saha and Ganguly [13] used a simple temporal estimate of the spatial density of the walkers. The temporal estimate proposed in [13] depends on the characteristics of the node-visits of a walker encountered within a certain temporal window (history). In this work we propose to modify this temporal estimate by assigning non-uniform weights to make the recent past visits more significant than visits made by the walker long ago. We find that this simple modification of the estimation of the history works better than the pure history-based solution (see figure 5).

On the other hand, a fundamental aspect of any graph search or traversal algorithm is the order the message follows to visit the vertices/nodes of the graph. Based on this aspect, the strategies can be classified either as breadth-first or depth-first. In a breadth-first strategy, starting from an initiator node, the traversal process first covers the immediate neighbours, then neighbours of neighbours and so on. The operation works level-by-level, the initiator being the first level. On the other hand, in a depth-first strategy, instead of exploring all the first-level neighbours initially, the process ideally tries to go deeper and deeper in the graph by exploring only a single node at each level. The process backtracks after it reaches the deepest level.

From this perspective the simple flooding algorithm can be viewed as a breadth-first strategy. Even the single random walk algorithm, which is inherently a probabilistic algorithm, can be understood as a depth-first strategy [14]. However, it is to be noted that, both the strategies have their own advantages and disadvantages. Thus, the applicability of any search and dissemination algorithm heavily depends on the kind of data being searched as well as the distribution of the data around the node initiating the process. In this paper we initiate a study in this direction and define circular coverage (around the initiator node) of nodes besides normal coverage. The results show that for a given energy constraint, there is a certain proliferation rate (hence, a specific time constraint) for which the achieved circular coverage is the best.

2. Background

Maximization of coverage in search or dissemination in networks has been the target of many works. Either variants of flooding [15–17] or single random walk [18–20] or their combinations (e.g., hybrid probabilistic schemes [21]) were used for this purpose. Different approaches have been adopted to minimize the redundant node visits by the message packet, e.g., use of topological features of the network [22,23] and entropy information measurement based cost effective path selection [24,25]. In order to meet the necessary time constraint, multiple copies of the message packets, i.e., multiple random walkers were introduced. The number of walkers were either fixed from the beginning to the end of the process [1,26–28] or were gradually increased in number following some specific rule throughout the process [29,30].

Thus, almost all the works in the direction of coverage maximization either try to best utilize the available energy or time or both by avoiding redundant visits of nodes, i.e., reducing the wastage of energy. A systematic study of the coverage achieved under a given constraint of time and energy were introduced first in [12,13] for two distinct regions of the problem space. Subrata Nandi *et al* [12] studied the region where the time constraint is more relaxed and proposed a time-varying proliferating RW strategy whereas Saha and Ganguly [13] considered a more practical region where the time constraint is stricter and proposed a history-based non-uniform proliferating RW solution.

On the other hand, different physical properties of RW as well as its variants have been extensively studied for many years by the physicists as well as the mathematicians. For example, the number of distinct sites visited by the walkers as well as the shape of the covered space by the large number of simultaneous multiple random walkers (initiated from a single node) were studied in [31–34]. Lawler [35] studied the probability of the intersection of random walk of length t in 4D. If these walks start from the same point, then the probability of no intersection is between quantities proportional to $1/\log t$ and $1/\sqrt{\log t}$.

As pointed out in the previous section, from the perspective of search and dissemination, depth-first or breadth-first property of any strategy happens to be an important issue. Single random walk has been studied as a simple depth-first searching tool in general random graph models [14]. However, a systematic study of these properties in a generalized platform is not available. In this paper we also focus on such a specific property of the RW-based strategies.

3. Metrics

The search and dissemination becomes most challenging in unstructured and decentralized networks. In unstructured networks there is no relationship between the information and the node identity where the information resides, whereas in decentralized networks there is no centralized control of the whole network, rather each node behaves independently and has only the information about the nodes which are directly connected with it. Hence, the underlying algorithms for such networks have non-deterministic and randomized components. The main aim of an algorithm is to spread message to as many nodes as possible. But due to its non-deterministic nature, some of the nodes may not

receive message properly. Moreover, due to the constraints on time and energy, messages cannot be passed indefinitely; the algorithms must stop after a certain amount of time has elapsed or after a certain amount of energy is spent in passing the messages. Thus, the processes achieve a partial covering of the network which is the subset of all the nodes in the network visited by the message packets. Given two partial coverings (corresponding two different instances of search/dissemination) of the network we have some intuition regarding which is more preferable in a particular situation. We need to model a situation by designing an appropriate metric for it. Such metrics will be useful in deciding the message passing strategy which needs to be followed in situations corresponding to the metric. We shall consider the following two classes of situations and design metrics for them.

3.1 Coverage

In situations where all the nodes are equally important with respect to search or dissemination, our metric is simply the number of covered nodes which is called the coverage.

3.2 Circular coverage

There could be situations in which all the nodes are not equally important with respect to the search or dissemination. In such situations, we associate weights (capturing the importance) with the nodes and measure the sum of the weights of the covered nodes instead of simply counting them. These weights will depend on the distance from the originating node. We consider a weighting scheme where the weights are inversely proportional to this distance. In this case, the metric will be the sum of the reciprocals of the distances of the covered nodes. We call this ‘circular coverage’ to remind us that if we draw a circle around the originating node, the nodes on the circumference get equal weightage.

The expected value of circular coverage of 1-RW depends on the total time T and will be used for comparison. At time t the walker is expected to be at a distance proportional to $\sqrt{t}$ from the origin. So the weight of the node visited at that time is expected to be proportional to $1/\sqrt{t}$. In 3D or higher dimensions, we know that a certain fraction (independent of t) of the visits will be redundant. This factor gets absorbed by the constant hidden in proportionality. So the expected circular coverage remains proportional to

$$\sum_{t=1}^{t=T} \frac{1}{\sqrt{t}} \approx \int_{t=0}^{t=T} \frac{1}{\sqrt{t}} \sim \sqrt{T}. \quad (1)$$

4. Strategy

There are two different covering problems. Both of them have two constraints for time (T) and energy (E). However, in one case the target is to maximize coverage and in the other it is for circular coverage. An optimization strategy fixes the rate of proliferation for each walker at each time with an aim to maximize the target metric. However, in this paper we primarily concentrate on the maximization of coverage and in parallel we also study its effect on the other metric. In the following, we present a step-by-step development of the concepts behind the optimization of the proliferation rates [12,13,36]. Finally, we propose a modification.

Without loss of generality, we assume that the number of initial walker is unity. If we have more than one walker initially, as at origin, we may interpret it as a single initial walker proliferating to achieve the desired number of initial walkers.

Time-based proliferation. If $T \geq C * E^{2/d}$, where $d \geq 3$ is the dimension and C is a constant with respect to T and E , Nandi *et al* showed that there is enough time to proliferate using the following rate and still expect the optimal coverage (that of 1-RW) [12]:

$$P^*(t) = \left(1 + \frac{1}{t + \xi - 1}\right)^{d/2-1} - 1.$$

In this equation, ξ is related to C . In regular grid they depend only on the dimension of the grid, $\xi \approx 80$ for $d = 4$ and $\xi \approx 150$ for $d = 3$.

But if the time is not enough, a higher proliferation rate is required to ensure that the energy quota is used before the time runs out. If $\alpha > 1$ and $x > 1$, then $x^\alpha > x$. Nandi *et al* used this expression to increase the rate of proliferation [12].

$$P^{(\alpha)}(t) = (1 + P^*(t))^\alpha - 1 \approx \alpha P^*(t).$$

History-based proliferation. Saha and Ganguly [13] showed that when T is much smaller than E , remembering a small sequence of nodes the walker has visited in the immediate past increases performance. From this sequence it can compute how many of them were self-intersections and how many of them were intersections with the trails of other walkers. Every node maintains a list of walkers visiting it. A walker entering this node can see this list to decide whether it is re-visiting a node and if so, whether it was a self-intersection or a mutual intersection.

Let ρ be the probability of making non-redundant visits by any particular walker during each of its visit in the near future. Clearly, ρ depends on how the neighbourhood of the walker is visited by all the other walkers (including itself) in the past. ρ will be high in a sparse region and low in a dense one. The rate of proliferation should be an increasing function of ρ so that walkers at sparser region are proliferated more. One way to do this would be

$$P^{(\alpha)}(t, \rho) = (1 + P^{(\alpha)}(t))^\rho - 1 \approx \alpha P^*(t)\rho.$$

If we want to bias the proliferation very strongly towards sparser regions, we can substitute $\rho^\gamma(h)$ instead of $\rho(h)$ for $\gamma \geq 1$. On the other hand, $\gamma = 0$ corresponds to being independent of history altogether. The modified rate of proliferation will be

$$P^{(\alpha, \gamma)}(t, \rho) = (1 + P^{(\alpha)}(t))^{\rho^\gamma} - 1 \approx \alpha P^*(t)\rho^\gamma.$$

Estimate of ρ . In the following, we first explain the way ρ is estimated by Saha and Ganguly [13] and then propose a modification on the strategy.

Let $(h_i)_{i=1}^{i=H}$ be a sequence of 0's and 1's depending on whether i steps ago the walker encountered a redundant visit or not, where H is the size of the history. Using this information, a node can estimate ρ as follows:

$$\hat{\rho}(h) = \frac{\sum_{i=1}^{i=H} h_i}{\sum_{i=1}^{i=H} 1} = \frac{\sum_{i=1}^{i=H} h_i}{H}.$$

We call it the ‘pure strategy’ and denote the corresponding rate of proliferation by $P(t, h)$ which is consistent with the introductory work done by Saha and Ganguly [13]. In this paper, we modify this pure strategy by considering the fact that a redundant visit which took place very recently should have more weight than the one which happened long ago. This new estimate can be achieved by selecting weights from a decreasing geometric series whose ratio is β . Here, $\beta \in (0, 1]$ will be called the ‘oldness factor’ in history. Thus, the modified estimate of ρ would be as follows:

$$\hat{\rho}(h) = \frac{\sum_{i=1}^{i=H} \beta^i h_i}{\sum_{i=1}^{i=H} \beta^i}.$$

We call it ‘weighted strategy’ and denote the corresponding rate of proliferation by $P(t, h')$ to remind us that it is a modification of $P(t, h)$. In the following section we study the effect of this modification by computer simulation of the corresponding processes.

5. Results

We begin this section by observing the variation in coverage as well as circular coverage of the RW-based strategies. Throughout this section the two measures are compared side-by-side. Then we move on to the coverage of K -random walk (K -RW). Later, we study the neighbourhoods of the random walkers during a K -RW process and observe the availability of sparse neighbourhoods. We then use the weighted history-based strategy to detect sparse regions in a decentralized way and proliferate there at a higher rate. We conclude this section with an apparent anomalous behaviour of the circular coverage by gradually decreasing the rate of proliferation to better utilize a gradually relaxing time constraint.

5.1 Variation in coverage of random walk strategies

A very straightforward observation is obtained from the distribution of the number of covered nodes by 1-RW in 2D. Theory says that in 2D the expectation of this number is $O(t/\log(t))$ [37] and its variance is $O(t^2/\log^4(t))$ [38]. From the histogram in figure 1a we can see that the constant hidden in O notation is quite high in case of variance. This implies that RW-based methods must be used with caution in critical situations. We also see that the distribution is neither very skewed nor very heavy-tailed.

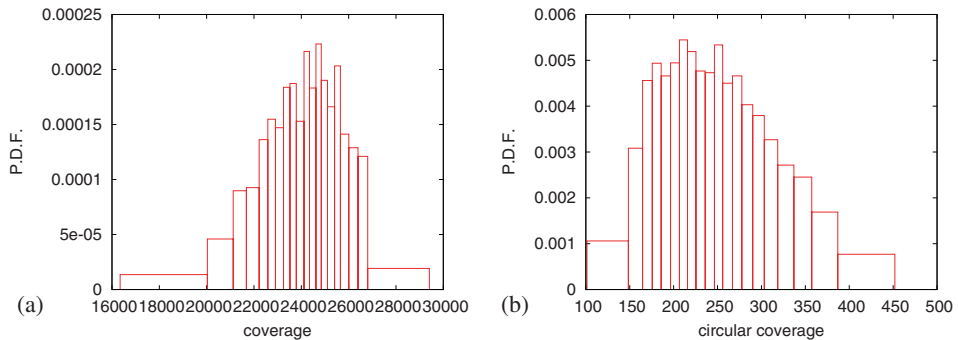


Figure 1. Histogram of the original and circular coverage of 1-RW when $E = 100,000$.

Figure 1b shows that the distribution of circular coverage has more variance (when normalized with respect to mean) than coverage. This could be explained from the fact that variance in coverage comes from one source of uncertainty, whether a visit at time t is redundant or not. However, in circular coverage one more source of uncertainty exists in case of new visit, the weight of it could be as high as $1/2$ and as low as $1/t$. We noted the skewness of the distribution as well. However, it does not look heavy tailed.

5.2 Coverages – K -random walkers

After studying the number of covered nodes we move on to visualize the set defined by them for $K = 1$ and 10 (figure 2). We kept the energy spent fixed to 100,000. It is observed that for some purposes the covering on the left is better than the right. The left one has covered more nodes than the right one. However, for some other purposes, covering a node closer to the originating node (at the centre of the snapshot) may be more useful than covering a distant one. In such cases, the right one is better. We can clearly see that even though the total number of nodes covered is higher on the left, the right one covers more nodes in the immediate neighbourhood around the origin.

Previous researchers have noted the maximum distance from the origin to a covered node and the minimum distance to an uncovered node [34], as well as the roughening of the boundary inside the annulus determined by these two radii [31].

After observing circularity we move on to measuring it while keeping the dimension same. We observe that the original measure of coverage as well as the circular measures are monotonically increasing function of time as well as energy. As K increases the original coverage decreases as some of the nodes are getting covered by more than one walker. This type of redundancy is called mutual overlap. In circular coverage, it initially increases with K as circularity increases with it. However, mutual overlap increases with K as well. There are two competing effects of increasing K and eventually for sufficiently large K the mutual overlap outweighs the increase in circularity. We also observe the eventual linearity of coverage from the fact that the walkers eventually reach regime III [31].

Figure 3 shows the plots of circular coverage (figure 3b) and plane coverage for a few values of k . We find that the plots of circular coverage fit well with the following equation:

$$G = a \times \sqrt{E},$$

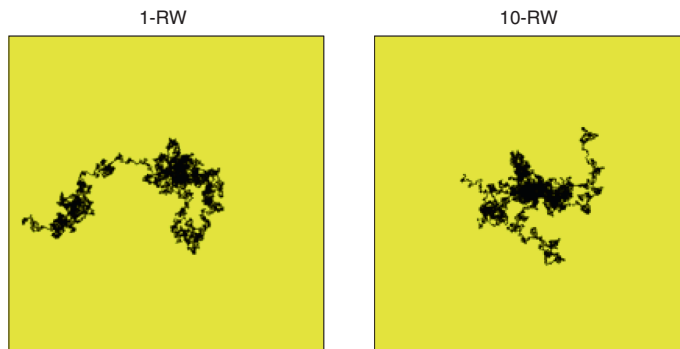


Figure 2. Snapshot of the nodes visited by 1-RW and 10-RW when $E = 100,000$.

where G and E denote the circular coverage and the energy consumed, respectively. It is to be noted that for K -RW, the value of E is just K times the total time consumed, because we measure E as the total number of messages passed in the system. a is a constant of proportionality. The values of a for different K are given in figure 3.

For small K , the circular coverage of K -RW in figure 3 seems proportional to $\sqrt{t}$ where the proportionality constant depends on K . Among the values of K studied, $K = 10$ gives the maximal coverage. If the dimension is 2 the fraction of non-redundant visit around time t is proportional to $1/\log t$. As $\log t$ changes much slower than $\sqrt{t}$, the integral $\int_{t=0}^{t=T} [1/\log(t)\sqrt{t}]$ will still remain approximately proportional to $\sqrt{T}$.

5.3 Availability of sparse regions in K random walk

After seeing what to measure we shall now focus on how to improve them. If we could detect the walkers in sparser regions we could improve our measures by increasing their rate of proliferation. However, we have to first ensure that such sparse regions are readily available. Saha and Ganguly detected sparsity by studying the number of walkers present

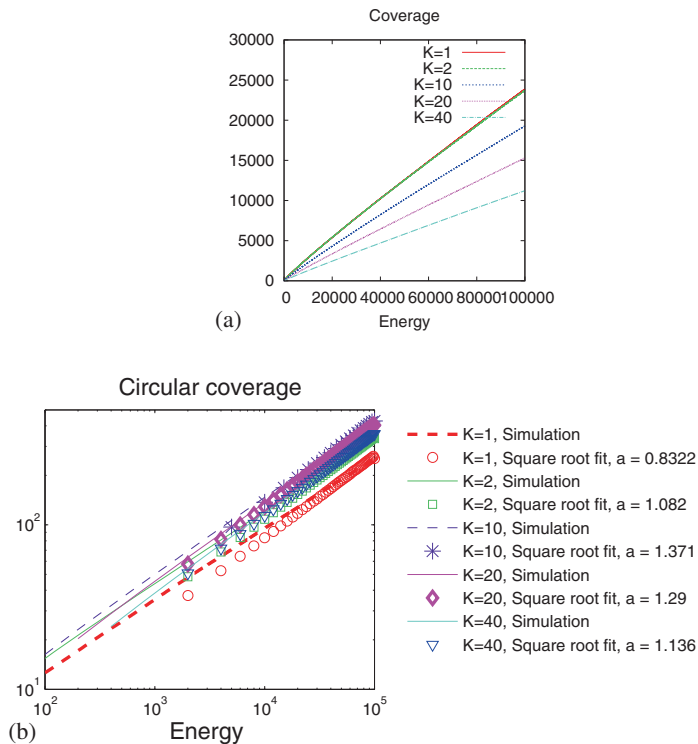


Figure 3. Total coverage (a) and circular coverage (b) of K -RW in 2D till $E = 100,000$. The circular coverage is plotted in log–log scale. The values of a for square root fitting for each plot are mentioned in the legend.

in cubes (in case of 3D) of volume 1000 [13]. We detect sparsity in a similar way, as described subsequently.

Suppose we want to detect a random walker in a sparse region after a message passing strategy has worked for some time. From its current position we allow it to take F steps in future, while we stop all the other walkers. We will detect sparsity by counting how many redundant visits it makes during those F steps. In a sparse region the expected fraction of non-redundant visits will be high. The contribution of the self-intersection during these F steps to this fraction is independent of the past and hence sparsity too is independent. So it can be ignored.

Our results show that in K -RW, even after consuming enough energy and large enough K , we do get walkers in sparse regions (figure 4a). We also see that the availability of the sparse regions does not change significantly even after doubling the consumed energy from 50,000 to 100,000 or K from 100 to 200 or F from 20 to 40.

From figure 4 we can also see that it is difficult to use the moderately sparse regions (region between (a) and (b)) because of high uncertainty (indicated by high standard deviation). This explains why we should take γ sufficiently high to intensely avoid redundant visits.

5.4 Coverages – weighted history-based proliferation

Motivated by our system study, we set $\gamma = 16$, $H = 20$ which is consistent with the previous history-based work done by Saha and Ganguly [13]. They reported that the improvement of performance due to history-based proliferation is most significant when T is small. For large T all proliferation strategies approach 1-RW and hence their performances are similar. Hence, we fix T at 200. The dimension and the upper limit of energy consumed are kept the same at 3 and 100,000, respectively. While keeping T fixed, we consume different amount of energy by varying α . When α is the same the consumed energy can vary a lot over an average of over 1000 runs. Similarly, we average

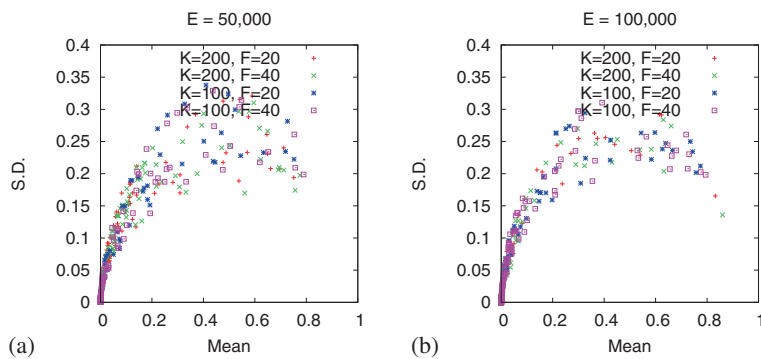


Figure 4. Sparsity being measured by the fraction of non-redundant visits in future F steps. Each plotted point corresponds to a walker. The x coordinate of the point is the expected value of the fraction and y coordinate is its standard deviation. Sparsity increases with x , uncertainty in sparsity measurement increases with y . The network is 2D grid and message passing strategy used is K -RW. E is the energy consumed by the time of sparsity measurement.

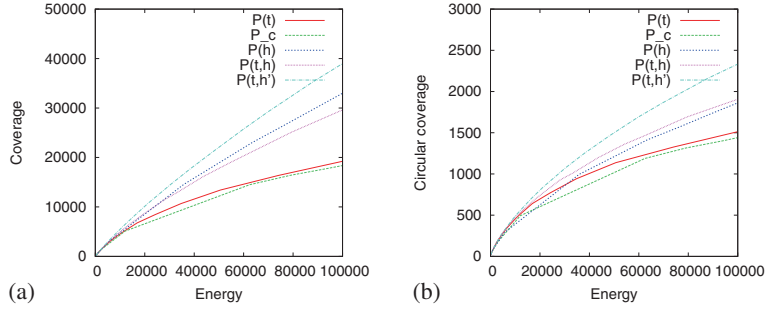


Figure 5. (a) History-based coverage and (b) circular coverage at $T = 200$.

the coverage (original and circular) for the same α . Then we plot the coverage as a function of consumed energy (figure 5). We see that weighted history $P(t, h')$ works the best and its relative improvement over the previously known best performing method ($P(h)$) is around 20%. Here we have used the oldness factor of history as $\beta = 0.8$. We have seen from experiments that varying this factor within $[0.7, 0.9]$ does not have any significant impact on performance. However, making β very close to 1 will lead to usual unweighted history-based proliferation and β close to 0 will be equivalent to having a history size (H) of 1.

5.5 Anomaly of circular coverage

The most interesting fact about circular coverage is seen in figure 6. The energy constraint (E) is fixed to 100,000. Dimension, γ , H are the same as in figure 5. The time constraint (T) is varied well below 200 to the value of E . When T is small the proliferation rate should be high which is controlled by α . Higher α leads to higher proliferation and to lower T . As T increases, the proliferation rate decreases and so does the mutual overlap among walkers. Decreasing mutual overlap leads to increasing coverage. As in the case of coverage, circular coverage continues to increase initially. But low rate of proliferation has a detrimental effect on circular coverage. As seen in figure 2 circularity of coverage increases with higher rate of proliferation (here K). In case of proliferation the covered nodes remain closer to the origin. Due to this, as the rate of proliferation drops beyond a limit, the advantage of lower mutual overlap becomes less than the disadvantage of higher average distance from the origin.

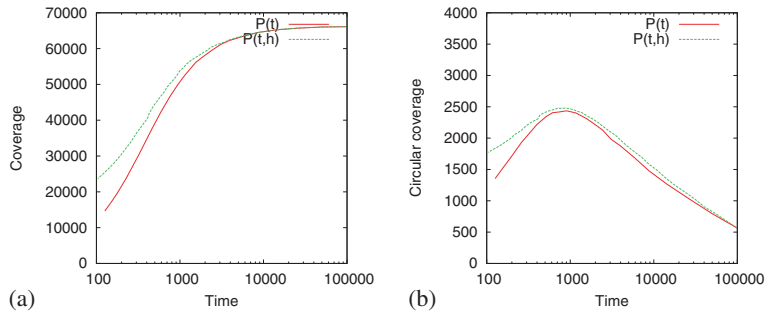


Figure 6. (a) History-based coverage and (b) circular coverage when $E = 100,000$.

6. Conclusion and future works

From our experiments (figure 5) we conclude that use of oldness factor in history is beneficial. This benefit however comes from the cost of having one more parameter in the rate of proliferation. The time needed to explore the space of parameters depends exponentially on the number of parameters. This suggests future work towards pruning the parameter space theoretically.

The metric circular coverage mainly indicates the breadth-first characteristics of a certain strategy. As seen in figures 3 and 6, 1-RW is certainly not optimal from this perspective. This leads to a fundamental question. In the absence of time constraint, which strategy optimizes circular coverage? The definition of circularity using weights inversely proportional to distance can be modified by considering other decreasing functions of the distance as well. How will the optimal strategy change if we do so? We expect such strategies to proliferate preferentially near the origin.

Given the energy and the time constraint, it is possible to theoretically estimate α only in the case of time-based proliferation. However, the uncertainty introduced by the availability of sparse regions makes it difficult to estimate α in history-based proliferation. This can be taken care of by searching for suitable α by trial and error. As the process has high variance we must average over a large number of runs to make this search for α systematic like a binary search. A theoretical estimate of α as a function of T and E will be very useful in history-based proliferation.

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