Noise, Decibel and Channel Capacity -18.01.2006

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1 Noise:

In any data transmission event, the received signal will consist of the transmitted signal plus additional unwanted signals that are inserted somewhere in between transmission and reception.The additional unwanted signals are called **noise**.

Noise may be divided into four categories:

- **Thermal noise:** This noise is due to thermal agitation of electrons. It is present in all electronic devices and transmission media and is function of temperature.This noise is uniformly distributed across the bandwidth. It is also called **white noise**. The amount of thermal or white noise in any device or conductor of bandwidth of 1 Hz is

$$N_0 = kT(W/Hz)$$

where

  - N_0 = noise power density in watts per 1 Hz of bandwidth
  - k = Boltzmann's constants = $1.38 \times 10^{-23} J/K$
  - T = temperature in Kelvins(absolute temperature)

This noise is independent of frequency. Thus thermal noise in watts present in a bandwidth of B Hertz can be expressed as

$$N = kTB$$

or, in decibel-watts,

$$N = 10\log k + 10\log T + 10\log B$$
$$= -228.6dBW + 10\log T + 10\log B$$

So if the bandwidth increase noise increases.

- **Intermodulation noise:**When signals at different frequencies share the same transmission medium, the result may be intermodulation noise. It is produced by nonlinearities in the transmitter,receiver, and/or intervening transmission medium. The effect of this noise is to produce signals at a frequency that is the sum or difference of the two original frequencies or multiple of those frequencies.

For instance :The mixing of signals at frequencies f_1 and f_2 might produce energy at the frequency $f_1 + f_2$. This derived signal could interfere with an intended signal at the frequency $f_1 + f_2$.

- **Crosstalk :**It is an unwanted coupling between signal paths.It can occur by electrical coupling between nearby twisted pairs or,rarely,coax cable lines carrying multiple signals.

  Crosstalk can also occur when microwave antennas pick up unwanted signals; although highly directional antennas are used, microwave energy does spread during propagation. It has been experienced by anyone who, while using the telephone, has been able to hear another conversation.

  Typically, crosstalk is of the same order of magnitude as, or less than thermal noise.

- **Impulse noise:**It is noncontinuous, consisting of irregular pulses or noise spikes of short duration and of relatively high amplitude. It is generated from a variety of causes, including external electromagnetic disturbance, such as lightning, and faults and flaws in the communication system.

  It is generally only a minor annoyance for analog data. For example, voice transmission may be corrupted by short clicks and crackles with no loss of intelligibility.However, it is the primary source of error in digital data communication. For example, a sharp spike of energy of 0.01 s duration would not destroy any voice data but would wash out about 560 bits of data being transmitted at 56 kbps. The digital data can be recovered from the signal by sampling the received waveform once per bit time.

2 Channel Capacity:

The maximum rate at which data can be transmitted over a given communication path, or channel, under given conditions, is referred to as the **Channel Capacity**. There are four concepts here that we are trying to relate one another:

- **Data rate:**This is the rate, in bits per second(bps), at which data can be communicated.
- **Bandwidth:**This is the bandwidth of the transmitted signal as constrained by the transmitter and the nature of the transmission medium. It is expressed in cycles per second, or Hertz.
- **Noise:**This is the average level of noise over the communication path.
- **Error rate:**This is the rate at which error occur, where an error is the reception of a 1 when a 0 was transmitted or the reception of a 0 when a 1 was transmitted.

2.1 Nyquist Bandwidth:

For noisless channel, the Nyquist bit rate formula defines the theoretical maximum bit rate C given by

$$C = 2 \times B \times \log_2 L$$

where

- B is the bandwidth of the channel
- L is the number of signal level used to represent the data
- C is the bit rate in bits per second(bps)

For example, Consider a noisless channel with a bandwidth of 3000 Hz transmitting a signal with two signal levels. The maximum bit rate can be calculated as:

$$B = 3000Hz, L = 2$$

so

$$\begin{aligned} C &= 2 \times 3000 \times \log_2 2 \\ &= 2 \times 3000 \\ &= 6000bps \end{aligned}$$

2.2 Shannon Capacity Formula:

In reality, we can not have a noisless channel; the channel is always noisy.

In 1944, Claude Shannon introduced a formula, called the Shanon's capacity, to determine the theoratical highest data rate for a noisy channel, given by

$$C = B \times \log_2 (1 + SNR)$$

where

- B is the bandwidth of the channel
- SNR is the signal to noise ratio i.e.

  $$SNR = \frac{S}{N}$$

  If we increase the signal strength then noise will also increase. We can express signal to noise ratio in decibel as

  $$SNR_{DB} = 10 \log \frac{S}{N}$$

- C is the Capacity of Channel in bits per second(bps)

For example, given bandwidth = 3 - 4 Hz, SNR_{DB} = 24 DB, Find the number of signal level that can be designed ?

Sol^n:

$$SNR_{DB} = 10 \log SNR$$

or

$$SNR = 10^{2.4}$$

now, by Shannon capacity formula,

$$C = 1 \times \log(1 + 10^{2.4})$$

$$= 2.4(neglecting)$$

applying Nyquist formula,

$$C = 2 \times B \times \log_2 L$$

or

$$2.4 = 2\log_2 L$$

or

$$L = 2^{1.2}$$

So the number of signal level, L = 2.

2.3 Decibels and Signal strength:

Signal strength is considered as the important parameter in any transmission system. As a signal propagates along a transmission medium, there will be loss, or *attenuation* of signal strength. Amplifiers may be inserted at various points to impart a gain in signal strength.

We express gains, losses and relative levels in decibels because

- Signal strength often falls off exponentially, so loss is easily expressed in terms of the decibel.
- The net gain or loss in a cascaded transmission path can be calculated with simple addition and subtraction.

The **decibel** is a measure of the ratio between two signal levels. The decibel gain is given by

$$G_{DB} = 10\log_{10}\frac{P_{out}}{P_{in}}$$

And decibel loss is given by

$$L_{DB} = -10\log_{10}\frac{P_{out}}{P_{in}}$$

$$= 10\log_{10}\frac{P_{in}}{P_{out}}$$

where

- G_{DB} = gain, in decibels
- L_{DB} = loss, in decibels
- P_{in} = input power level
- P_{out} = output power level