Assignment - Reuse Factor in Wireless Networks

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The Assignment Problem is to Prove that

$$N = i^2 + j^2 + i * j; i, j = 0, 1, 2, 3, ... \tag{1}$$

and

$$D/d = \sqrt{N}. \tag{2}$$

where, D = distance between centers of the nearest co-channel cells.
d = distace between centers of adjacent cells
N = cluster size

Theory
Consider a cellular system which has a total of S duplex channels available for use. If each cell is allocated a group of k(k¡S) channels , and if the S channels are divided among N cells into unique and disjoint channel groups which have the same number of channels , the total number of available radio/wireless channels can be expressed as ,

$$S = kN \tag{3}$$

The N cells collectively use the complete set of available frequencies is called a cluster.
The factor N is called the cluster Size. The frequency Reuse factor is given by 1/N , since each cell within a cluster is assigned 1/N of the total channels in the system.

We can clearly see that ,

$$R = d/\sqrt{3} \tag{4}$$

where,

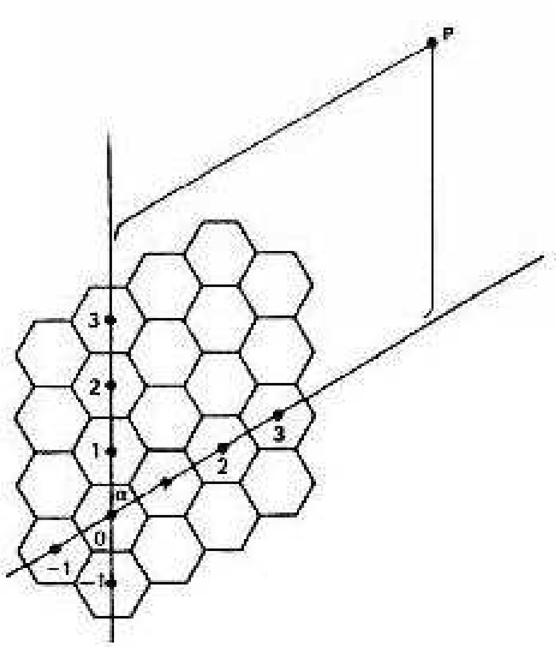

Figure 1: The geometry in the Hexagonal cellular system

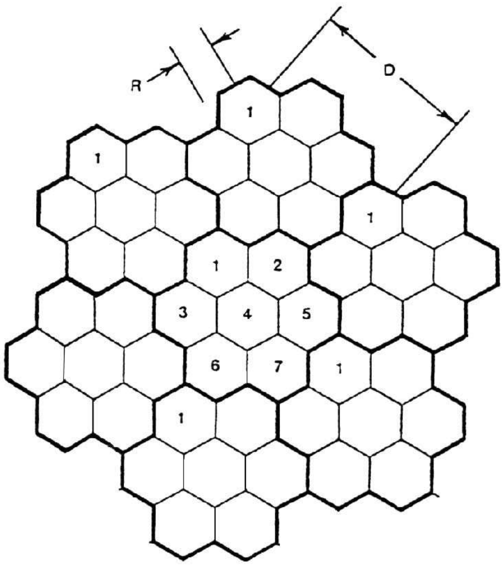

Figure 2: The geometry in the Hexagonal cellular system

R = radius of one hexagonal cell.
d = distace between centers of adjacent cells
Now , Lets assume that i be the distance along the chain of hexagons and j be distance moved after turning through 60 degrees counter-clockwise. Then, the distance between centers of the two hexagons can be given by the formula

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)(x_2 - x_1) + (y_2 - y_1)^2} \tag{5}$$

where, (x1,y1),(x2,y2) are the co-ordinates of the centres of the two hexagons in a Co-ordinate System with two axes at an angle of 60 degrees (not perpendicular).
So, by putting i,j into place ,we get

$$D = d * \sqrt{(i^2 + ij + j^2)} \tag{6}$$

Now, Area of one Hexagon is given by

$$Area = \sqrt{3}/4 * R^2 \tag{7}$$

where, R is the radius of hexagon.
Using an Approximate Heuristic , we can say that each cluster will follow the hexagon pattern with radius

$$D/\sqrt{3} \tag{8}$$

$$ArC = \sqrt{3}/4(D/\sqrt{3})^2 \tag{9}$$

where, Total Area of the a cluster = ArC
Therfore, Number of Cells in a Cluster, N is given by N = Area of Cluster / Area of one Cell

$$N = (D/\sqrt{3}R)^2 \tag{10}$$

Thus,

$$D/R = \sqrt{3N} \tag{11}$$

$$N = (D/R\sqrt{3})^2 \tag{12}$$

$$N = (D/d)^2 \tag{13}$$

$$N = i^2 + ij + j^2; i, j = 0, 1, 2, 3, ... \tag{14}$$

Hence , Proved !!!