Assortatively and its effect on Community Structure

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1 Assortatively

Out of several different possible explanation of the formation of communities in the social network, assortatively is most prominent. It states "rich goes with rich".

1.1 Example

Consider the following men-women marriage relation depending on their race.Here (i, j) element of the matrix gives the fraction of men of i^{th} race marry women of j^{th} race.

	black	hispanic	white	others
black	0.258	.016	.035	.013
hispanic	.012	.157	.058	.019
white	.013	.023	.306	.035
Others	.005	.007	.024	..016

Diagonal elements represent the fraction of couples in partnership with members of their own group and off-diagonal those in partnership with members of other group. Inspection of the matrix shows that matrix has the matrix has considerably more weight along the its diagonal than off it indicating that assortative mixing does take place. The amount of assortative mixing in a network can be quantified by measuring how much of the weight in the mixing matrix falls on the diagonal and how much off it. Let us define e_{ij} to be the fraction of all edges in a network that joins the vertex of type i with type j. According to the matrix, we can say that index i represents man and j represents female. which makes e_{ij} asymmetric. The matrix should satisfy the sum $\sum_{ij} e_{ij} = 1$,$\sum_j e_{ij} = a_i$ and $\sum_i e_{ij} = b_i$ where a_i and b_i are the fraction of each type of end that is attached to vertices of type i. Now we define a quantitative measure r of the level of assortative mixing in the network,

$$r = \frac{\sum_i e_{ii} - \sum_i a_i b_i}{1 - \sum_i a_i b_i} \tag{1}$$

It takes the value 1 for the perfectly assortative network, since in that case, the entire weight of the matrix lies along its diagonal. Conversely, if there is no assortative mixing at all, then r becomes quite low. Networks can also be disassimilative: vertices may associate preferentially with others of different types - the opposite attracts phenomenon. In that case, r becomes negative.

Given a mixing matrix of the type shown in above table, we can create a random network with the corresponding mixing pattern and any desired degree distribution by the following algorithm. Algorithm to synthesis a network from the given assortative mixing pattern is given below

1.2 Network Synthesis Algorithm

Step 1: Choose the degree distribution $p_k^{(i)}$ for each type vertex i. The quantity $p_k^{(i)}$ denotes the probability that a randomly chosen vertex of type i will have degree k. We can calculate the mean degree $z_i = \sum_k k p_k^{(i)}$. for each vertex type.

Step 2: Next we choose the size of the network in terms of the number m of edges. and draw m edges from the desired distribution e_{ij}. We count the number of ends of edges of each type i to give the sums m_i of degrees of vertices in each class and we calculate the expected number n_i of vertices of each type from $n_i = m_i/z_i$.

Step 3: We draw n_i vertices from the desired distribution $p_k^{(i)}$ for type i. Normally the degrees of these vertices will not sum exactly m_i in which case we choose one vertex at random, discard it and draw another from the distribution $p_k^{(i)}$, repeating until the sum equals m_i.

Step 4: We pair up the m_i ends of edges of type i at random with the vertices we have generated, so that each vertex has the number of attached edges corresponding to its chosen degree.

Step 5: We repeat from step 3 each vertices.

2 References

1. "Mixing patterns and community structure in networks", M. E. J. Newman and M. Girvan, in Statistical Mechanics of Complex Networks, R. Pastor-Satorras, J. Rubi, and A. Diaz-Guilera (eds.), Springer, Berlin (2003).
2."Assortative mixing in networks", M. E. J. Newman, Phys. Rev. Lett. 89, 208701 (2002).