Generalized Random Graph

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1 Introduction

Generalized random graphs [1] have arbitrary probability distribution of the degrees of their vertices. In all respects these graphs are assumed to be entirely random. That is the degrees of all the vertices are independent identically distributed random integers drawn from a specified distribution. Any given choice of the degrees of the vertices are represented as *degree sequence*. For construction of the random graph first specify the *degree sequence* of the vertices. We can consider any vertex as a node having spokes as the degree. Now randomly choose 2 spokes and join them by an edge. So, there is a possibility of having *self loop* and *parallel edges*. Here in this model we consider the ensemble of the graphs generated in this way.

2 Generating Function

We are considering the *generating function* $G_0(x)$ for the probability distribution of vertex degrees k. We define
$G_0(x) = \sum_{k=0}^{\infty} p_k x^k$,
where p_k is the probability that a randomly chosen vertex on the graph has degree k and x is a particular property of the node. For example x can be the *degree* of the node. Here, $G_0(1) = \sum_{k=0}^{\infty} p_k = 1$.
Given $G_0(x)$ we have to find p_k.
$\frac{dG_0(x)}{dx} = p_1 + 2p_2 x + 3p_3 x^2 + \ldots + kp_k x^{k-1}$

Similarly,
$\frac{d^kG_0(x)}{dx} = k(k-1)(k-2)\ldots 1p_k + (k+1)k\ldots 2p_{k+1}x + \ldots$
$\Rightarrow \frac{d^kG_0(x)}{dx}|_{x=0} = k!p_k$
$\Rightarrow p_k = \frac{1}{k!}\frac{d^kG(x)}{dx}G(x)|_{x=0}$

2.1 Moments

The average degree z of a vertex i.e. the *first moment* in the case of $G_0(x)$ is given by
1 *st moment* $z = \sum_{k=0}^{\infty} kp_k$
$= \frac{dG_0(x)}{dx}|_{x=1}$
Similarly,
n th moment$=\sum_{k=0}^{\infty} k^n p_k$
$=(x\frac{d}{dx})^n G_0(x)|_{x=1}$

References

[1] "Random graphs with arbitrary degree distributions and their applications", M. E. J. Newman, S. H. Strogatz, and D. J. Watts, Phys. Rev. E 64, 026118 (2001).