Complex Network Theory - 12.01.2006

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I. Graph Theory

A. Diameter of the graph

Diameter of the graph is maximum shortest distance between any two nodes in the graph.

First find shortest distance between two vertices. A is the adjacency matrix.

A^r[i,j]: number of r-hop distant paths from vertex i to j.

A^1[i,j], A^2[i,j],.... A^r[i,j],...

Find the smallest r for which A^r[i,j] is non-zero. Then r is the shortest distance between vertices i and j.

(vi,vj) = total n(n-1)/2 pairs. n is the no. of nodes.

We go on adding the matrices and at some time all the elements in the matrix become +ve or equal to 1, we stop at that point and that is the diameter of the matrix.

Diameter D = r where $\sum_{k=1}^{r} A^k$ is all +ve while $\sum_{k=1}^{r-1} A^k$ is not.

Shortest Path SP(i,j) = r where A^r[i,j] is +ve and For All s<r A^s[i,j]=0.

Shortest Path SP(i,j) = r where $\sum_{k=1}^{r} A^k[i,j]$ is +ve and $\sum_{k=1}^{r-1} A^k[i,j]$ is not.

If $\sum_{k=1}^{n} A^k[i,j]$ =0 for at least one position(i,j) then it is a disconnected graph, where n = number of vertices in the graph.

Diameter of a disconnected graph is infinity.

Maximum possible value of the diameter of a connected graph = n-1.

B. Directed Graph

Citation network is strictly directed grpah.

Friendship network is directed graph but not strictly directed.

Adjacency matrix for directed graph need not be symmetric.

Directed graph can have Strongly Connected Components and Weakly Connected Components

For undirected graph SCC and WCC are same.

For directed graph: Remove the links and make them undirected edge i.e. undirected graph.

If both the directed and undirected graphs are connected then it is strongly connected component.

If only undirected graph is connected, it is a weakly connected component.

If none of them are connected, it is disconnected.

Disconnected Graph: $A = \begin{pmatrix} A1 & 0 \\ 0 & A2 \end{pmatrix}$

Weakly Connected Graph: $A = \begin{pmatrix} A1 & A3 \\ 0 & A2 \end{pmatrix}$

OR, $A = \begin{pmatrix} A1 & 0 \\ A4 & A2 \end{pmatrix}$

Strongly Connected Graph: $A = \begin{pmatrix} A1 & A3 \\ A4 & A2 \end{pmatrix}$

C. Incidence Matrix

The following is an incidence matrix for a graph consisting of 6 vertices and 4 edges. Edge e1 connects vertices v4 and v5, edge e2 connects vertices v1 and v2 and so on.

	e1	e2	e3	e4
v1	0	1	1	0
v2	0	1	0	0
v3	0	0	0	1
v4	1	0	0	0
v5	0	0	1	1
v6	1	0	0	0

With Incidence Matrix we can't represent self loop but can represent parallel edges.

Parallel edges are important for routing (transportation network).

Self loop has a degree of 2.

Condition for Disconnectivity (iff): $I = \begin{pmatrix} I1 & 0 \\ 0 & I2 \end{pmatrix}$

Where I1 and I2 are incidence matrices for the two disjoint subgraphs.

Rank(I) is n-1. I is the incidence matrix.

Rank is number of independent linear equations.

Because nth row can be derived from (n-1) rows by adding columnwise and taking modulo 2.