Complex Network Theory - 06.01.2006

Kamal Kumar Sultania, 01CS3004

I. Graph Theory

Started by Euler.

First Problem of Graph Theory: Konigsberg Brucke Problem. Which means king's palace bridge problem.

Traverse all the bridges exactly once and come back to the same point.

Consider the islands as nodes.

Euler tour(closed walk): Traverse all the edges exactly once and come back to the same point.

Euler walk(open walk): Traverse all the edges exactly once and need not come back to the same point.

Theorem 1: A graph has a Euler tour(closed walk) iff all the nodes have even degree.

Proof: Suppose a node has odd degree then, Entry != Exit and hence it can not have a Euler tour. ■

Theorem 2: A graph has Euler walk(open walk) iff 2 nodes have odd degree.
In this case, you can find a Euler walk starting from an odd degree node and end on another odd degree node.

A. Unicursal Line (Edge disjoint subgraphs)

Number of nodes with odd degree = even.

Graph G has 2k-odd degrees.

We can divide the graph G into k number of edge-disjoint subgraphs.

1. 2k number of nodes with odd degrees
2. Take these 2k nodes pairwise (k pairs)
3. Draw edge between each pair
4. Go on removing these k edges, we will get k edge-disjoint subgraphs which will have Eulerian open walk.

Odd number of nodes having odd number of degrees is not possible.

II. Graph Theory and Matrices

Representation of a graph: Adjacency Matrix.

For example, following is an adjacency matrix for a graph of 5 vertices, where node 1 has a self loop, node 2 is connected to itself and node 3 and so on.

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

By Adjacency Matrix we can't represent parallel edges, but we can represent self loop.

Converting into a matrix helps prove many problems because Linear Algebra has many standard results.

Many isomorphic relations by row-column transformations/operations.

For undirected graph adjacency matrix is symmetrical.

Easy to find disjoint subgraphs.

For example, if T is the adjacency matrix then, nodes (1), (2,3) and (4,5) are forming disjoint subgraphs.

$$T = \begin{pmatrix} \lceil 1 \rfloor & 0 & 0 & 0 & 0 \\ 0 & \lceil 1 & 1 \rceil & 0 & 0 \\ 0 & \lfloor 1 & 1 \rfloor & 0 & 0 \\ 0 & 0 & 0 & \lceil 0 & 1 \rceil \\ 0 & 0 & 0 & \lfloor 1 & 1 \rfloor \end{pmatrix}$$

Powers of X(Adjacency Matrix)

$$X^k = X * X * ....ktimes \tag{1}$$

Xij^2 represents the number of paths of length 2 from vertex i to j.

Xij^k represents the number of k-hop paths from vertex i to j.

$$Xij^2 = \sum_{k=1}^{n} Xik * Xkj \tag{2}$$