We know that the ordered degree mixing is different from other type of mixing.

Edge matrix

	1	2	3	4
1	e_{11}	e_{12}	e_{13}	e_{14}
2		e_{22}		
3			e_{33}	
4				e_{44}

We can say that an edge connecting degree 2 and degree 3 nodes contribute more towards the assortativity than the edge connecting degree 1 and degree 4.

Let us include the concept of remaining degree distribution. We follow an edge and reach a node with degree K. Since this edge is connected to one of its degree, therefore there are K-1 degrees remaining.

$\therefore$ $Q(K)$ = Probability of finding a node with K remaining degree

$$= \frac{\text{One tip of any } (K+1) \text{ degree node}}{\text{Total tips}}$$

$$= \frac{(K+1) \times N \times P(K+1)}{\sum_j j P(j) \cdot N}$$

$$\Rightarrow \boxed{Q(K) = \frac{(K+1)\, P(K+1)}{\sum_j j \cdot P(j)}} \quad \text{— 1 (a)}$$

When we say $K Q(K)$, ~~it~~ it is nothing but

$$\frac{(K+1)(K+1)\, P(K+1)}{\sum j \cdot P(j)} \quad \text{— 1 (b)}$$

2.

Let us define ~~$\varphi(k, k')$~~ $\varphi(j,k)$ which is the probability of finding an edge such that one of the end of the edge contains j remaining degree and another end contains k remaining degree.

It can be written as

$$\boxed{\varphi(j,k) = P(j+1, k+1)} \quad \text{--- (2)}$$

In a network where remaining degree is independent-

$$\boxed{\varphi(j,k) = \varphi(j)\cdot\varphi(k)} \quad \text{--- (3)}$$

In case of ~~highly~~ assortative graph

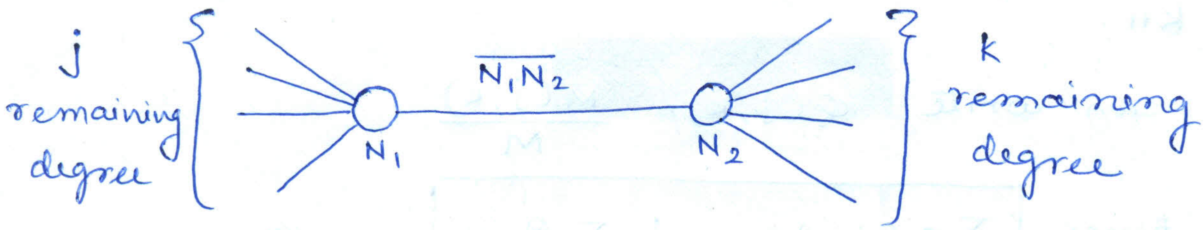
$$\cancel{\varphi(j,k) = \varphi(k)\cdot\varphi(j)\cdot\delta[k]}$$

$$\boxed{\varphi(j,k) = \varphi(j)\cdot\delta[j-k]} \quad \text{--- (4)}$$

Here, every edge exists due to the assortativity. And every edge produces an assortativity of

$$\boxed{\varphi(j,k) - \varphi(j)\cdot\varphi(k)} \quad \text{--- (5)}$$

Let us consider an edge between a node of degree $j+1$ and a node of $k+1$ degree.



The edge $\overline{N_1N_2}$ produces assortativity of $\varphi(j,k) - \varphi(j)\cdot\varphi(k)$.

But this edge can be reaced using any of the j remaining degree of N_1 or any of the k remaining degree of N_2.

$\therefore$ the total amount of assortativity that is contributed by the edge N_1N_2 is given as

$$\boxed{jk\left[\varphi(j,k) - \varphi(j)\cdot\varphi(k)\right]} \quad \text{— (6)}$$

$\therefore$ assortativity contributed by all the edges in the graph

$$\boxed{\sum_{j,k} j\cdot k\cdot\left[\varphi(j,k) - \varphi(j)\cdot\varphi(k)\right]} \quad \text{— (7)}$$

By normalizing eq. (7) we can write assortativity coefficient as

$$\boxed{r = \frac{\sum_{j,k} j\cdot k\cdot\left[\varphi(j,k) - \varphi(j)\cdot\varphi(k)\right]}{\sum_{j,k} j\cdot k\cdot\left[\varphi(j)\cdot\delta[j-k] - \varphi(j)\cdot\varphi(k)\right]}} \quad \text{— (8)}$$

Let M be the total no. of edges.

Let M(j,k) represents no. of links between degree j+1 and degree k+1.

∴ we can write $q(j,k) = \frac{M(j,k)}{M}$

and hence $\boxed{\sum_{j,k} q(j,k) = \frac{1}{M} \sum_{j,k} jk}$ — (9)

First term of the denominator of the eq. (8) can be written as

$$\sum_{j,k} jk\, q(j) \cdot \delta[j-k]$$

$$= \sum_j j^2 q(j) \qquad [\because \delta[j-j]=1 \text{ for } j=k]$$

$$\cancel{= \frac{\sum_j (k+1)^3 \cdot P(k+1) \cdot N}{\sum_j j \cdot P(j) \cdot N}}$$

$$= \frac{\sum_j (j+1)^3 P(j+1) \cdot N}{\sum_j j \cdot P(j) \cdot N}$$

$$= \frac{\sum_j j^3}{2M} \quad \text{------ (10)}$$

The second term in both numerator and denominator

$$\sum_{j,k} jk\, \varphi(j)\cdot\varphi(k) = \left[\sum j\cdot\varphi(j)\right]^2$$

$$= \left(\frac{\sum_j (j+1)^2 \cdot P(j+1)\cdot N}{\sum_j j\cdot P(j)\cdot N}\right)^2$$

$$= \left(\frac{\sum_j j^2}{2M}\right)^2 \quad \text{----} (11)$$

Using the values from eq. 9, 10 and 11 we can rewrite the eq. 8 as

$$\boxed{r = \frac{\frac{1}{M}\sum_{j,k} jk - \left(\frac{\sum_j j^2}{2M}\right)^2}{\frac{\sum_j j^3}{2M} - \left(\frac{\sum_j j^2}{2M}\right)^2}} \quad \text{----} (12)$$

Li et. al. in 2005 showed that this is equivalent to Newman's assortativity coefficient,

$$r = \frac{\frac{1}{M}\sum_{j,k} jk - \left(\frac{1}{2M}\sum_{j,k}(j+k)\right)^2}{\frac{1}{2M}\sum_{j,k}(j^2+k^2) - \left(\frac{1}{2M}\sum_{j,k}(j+k)\right)^2}$$