# A Game Theoretic Framework for Incentives in P2P Systems

Gyan Baboo (07CS3018)
Naveen Kumar (07CS1008)
Varun Mundra (07CS1023)

# Outline

1. Introduction (problem, motivation)
2. Incentive model
3. Nash Equilibrium in Homogeneous Systems of Peers
4. Nash Equilibrium in Heterogeneous Systems of Peers
5. Simulation result
6. Summary

## Introduction

As we know P2P systems have democratic nature, which means there is no central authority to mandate or coordinate the resource that each peer contributes. Because of voluntary participation, the distributed resources are highly variable and unpredictable. Recent research shows that many users are simply consumer and do not contribute much to the system. In particular, they found that user sessions are relatively short; 50% of the sessions are shorter than 1 hour, and many users are free riders that is, they contribute little or nothing. Short session means that a large portion of the data in the system might be unavailable for large period of time. When the growing number of free riders, the system starts to loose the sprit of peer to peer and becomes a traditional client server system.

## How to build a reliable P2P system

If the peer to peer systems are to become a reliable platform for distributed resource sharing, they must provide predictable level of service. So, in order to let peer makes contribution as much as possible, there are two economic methods that can be used monetary payment needs a imaginary currency and requires a accounting infrastructure to track various resource transaction and charges for them using micro payments. But as written in the paper, it is highly impractical because of network pricing Reputation reflects the overall contribution to the system

## Modeling interaction of peers by Game Theory

Because all peers are strategic and rational player, it is very intuitive to model the interaction of peers in game theory.
We say player are strategy players because users compete for shared but limited resources and at the same time they restrict others download from their server by deny access or not contribute anything.
So it is actually a Non-cooperative game among peers: each player wants to maximize his utility. Utility is a notion from game theory.Utility depends on benefit and cost. Utility depends not only on his own strategy but everybody else's strategy.From a peer's point of view, he might decide to unilaterally switch his strategy to improve his utility. This switch in strategy will affect other players'utility and they might decide to switch their strategy as well. So you see it is dynamic iterative process. The collection of players is said to be at **Nash equilibrium** if no player can improve his utility by unilaterally switching his strategy. So it is converged.

## Incentive model (measure contribution)

Before we start to find Nash equilibrium, it is necessary to define some notion.

- $P_1, P_2, P_3 \ldots P_N$ as peers
- Utility function for $P_i$ is $U_i$
- Contribution of $P_i$ is $D_i$ ($D_0$ is absolute measure of contribution)

$$d_i \equiv D_i / D_0$$

- Dimensionless contribution:
- Unit cost $c_i$
- Total cost: $c_iD_i$

## Incentive model (Benefit matrix)

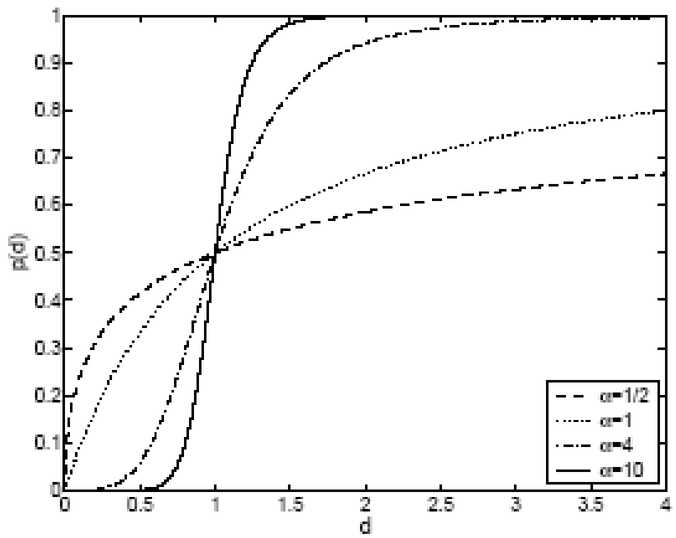
Each peer's contribution to the system will potentially benefit all other peers, but perhaps to different degree.
In order to express this different degree, an NxN benefit matrix B is introduced. $B_{ij}$ denotes how much the contribution made by $P_j$ is worth to $P_i$ (measured in dollars)
$B_{ij}$ = 0 means i not interested in j's contribution.
$b_i$ is the total benefit that $P_i$ can get from the system

$$b_{ij} = B_{ij} / c_i$$
$$b_i = \sum_j b_{ij}$$
$$b_{av} = \frac{1}{N}\sum_i b_i$$

We shall show that there exists a critical value of benefit $b_c$ such that if
$b_i < b_c$, then $P_i$ is better off not joining the system.
$b_{av}$ is the average of benefit($b_i$) for the whole system.
In case of differential service (game of expectations) a peer rewards other peers in proportion to their contribution. A simple approach to implement this idea is to say $P_j$ accepts a request for a file from peer $P_i$ with probability $p(d_i)$ and rejects it with probability $1-p(d_i)$ So, if Pi's contribution is small, its request is more likely to be rejected. Each request is tagged with $d_i$ as metadata. We use this probability function, actually any probability function is ok if it is a monotonically increasing function of the contribution

The function written below has nice properties that when contribution is 0, the probability the other peer accepts the request is 0
When contribution is infinitive, then the others peers are most likely to accept his request
Incentive model (A peer reward other peers in proportion to their contribution)

$$p(d) = \frac{d^\alpha}{1+d^\alpha}, \alpha > 0$$
$$p(0) = 0$$
$$\lim_{d\to\infty} p(d) = 1$$




## <u>Incentive model (Utility function)</u>

- Utility function

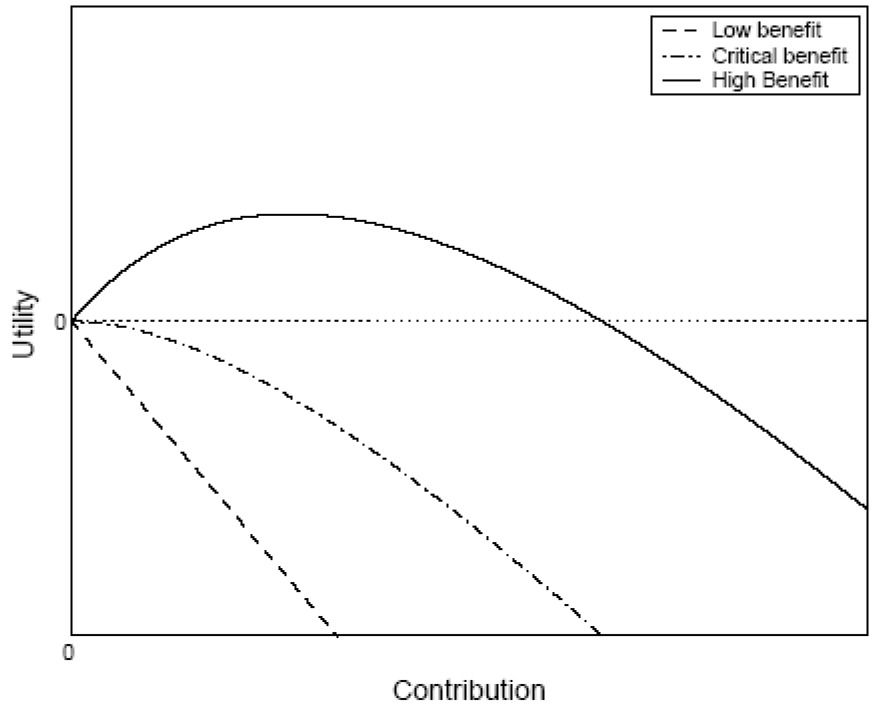
$$U_i = -c_i D_i + p(d_i)\sum_j B_{ij} D_j , B_{ii} \equiv 0$$

- Dimensionless utility function

$$u_i = \frac{U_i}{c_i D_0}, u_i = -d_i + p(d_i)\sum_j b_{ij} d_j , b_{ii} \equiv 0 \quad \text{.........................(A)}$$

The first term is the cost to join the system, while the second term is the total expected benefit from joining the system.
After we do normalization, we get Dimensionless utility function $u_i$.
The first term is Pi's cost to join the system and it increases linearly as peer contributes more disk/bandwidth to the system. Pi's benefit depends on how much the other peers are contributing to the system (dj ), how that contribution is worth to him ($b_{ij}$), and the probability that he is able to download that content p(di) or in other words, whether the other would like to accept his request.

## Utility vs. contribution (different benefit)




This figure show different utility function for different level of benefit.
Unless there is a critical value bc, the utility is always less than 0.

## Homogeneous System of Peers

We define a homogeneous system of peers to be a system where $b_{ij}$ = b for all $i \neq j$
In the homogeneous system, the model of equation A reduces to

$$u = -d + (N-1)bdp(d)$$

In a homogeneous system of two players, Equation 6 reduces to

$$u_1 = -d_1 + b_{12}d_2 p(d_1)$$
$$u_2 = -d_2 + b_{21}d_1 p(d_2)$$

If we take p(d) as following

$$p(d) = \frac{d^\alpha}{(1+d^\alpha)}, (\alpha = 1)$$

We expect that if the benefits that the peers derive from each other, i.e. b12 and b21 are too small then it will be best for the peers not to join. The question to ask at this point is whether a Nash equilibrium exists for large enough values of benefits where both peers can derive non-zero utility from their interaction.

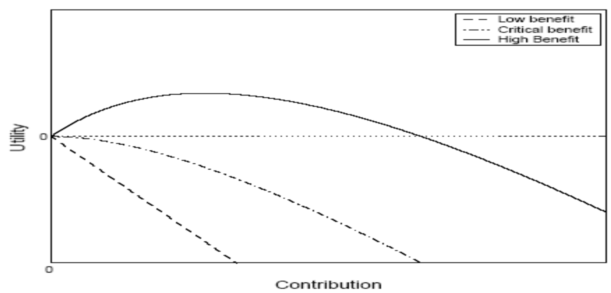
Suppose $P_2$ decides to make a contribution $d_2$ to the system. Given this contribution $d_2$, naturally the best thing for $P_1$ to do is to tune his $d_1$ such that it maximize his utility $u_2$. Maximizing u1 with respect to d1, we immediately find that the best response d1 is given by

$$r_1(d_2) \equiv d_1 = \sqrt{b_{12} d_2} - 1$$
$$r_2(d_1) \equiv d_2 = \sqrt{b_{21} d_1} - 1$$

Similarly $d_2$ can be obtained.

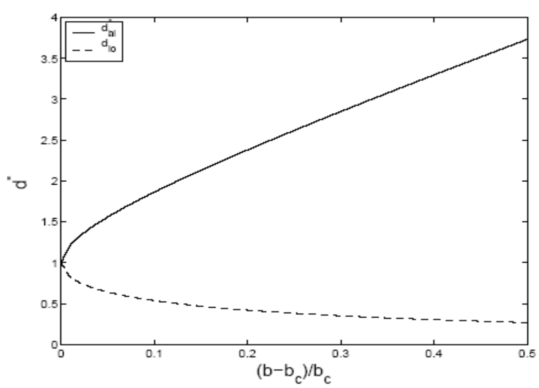
Nash equilibrium 1 exists if there is a set of $(d_1^*, d_2^*)$ such that they form a fixed point for above equations i.e.the fixed points satisfy

$$d_1^* = d_2^* = d^*$$
$$d^* = (\frac{b}{2} - 1) \pm \sqrt{(\frac{b}{2} - 1)^2 - 1} \quad \text{(b>=4)}$$




The Nash equilibrium contributions for the two peer system plotted as a function of scaled benefit $(b - b_c)/b_c$. For $b < b_c$, there are no equilibria. For all $b > b_c$ there are two possible equilibria.

## Critical benefit value $b_c$




For

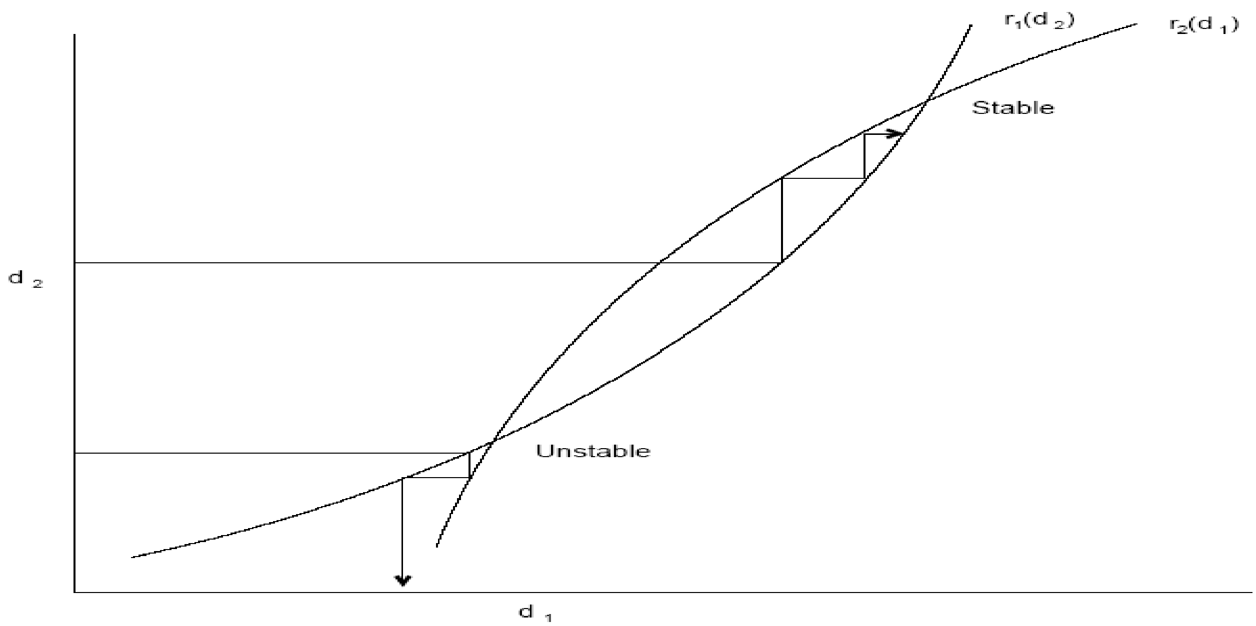
$b = b_c$, the only solution is $d_1 = d_2 = 1$.For $b < b_c$, there are no equilibrium. For all $b > b_c$ there are two possible equilibrium.

## Nash Equilibrium in Homogeneous System of Peers(N player game)

- **Replace b(N-1) to b, this formula is two player game.**

$$d^* = \sqrt{b(N-1)d^*} - 1$$
$$d^* = \left(\frac{b(N-1)}{2} - 1\right) \pm \sqrt{\left(\frac{b(N-1)}{2} - 1\right)^2 - 1}$$

## Courtnot learning & convergence process




As we see we have two solutions here. Now the question is which Equilibrium will be choosed by the system in practice.

Suppose the user $P_2$ sets his contribution to some $d_2$ to start with. In this situation, $P_2$ can use the reaction function $r_1(d_2)$ to set his optimum contribution at $d_1$. Seeing this contribution $P_1$ adjusts his own contribution and each peer takes turns in setting their contribution. If this process converges, then naturally that level of contribution for $P_1$ and $P_2$ will constitute a Nash equilibrium.

From the figure we see that under this learning process, either the peers will quit the game (zero utility) or they will converge to the equilibrium $d_{hi}$.

$$d_1^* = \sqrt{b_{12}d_2^*} - 1$$
$$d_2^* = \sqrt{b_{21}d_1^*} - 1$$

## Nash Equilibrium in Heterogeneous System

In a heterogeneous system, we need to deal with the full complexity of the model. The fixed point equations for $\alpha = 1$ can be immediately derived in analogy with the two player game as alterative learning model

$$d_i^* = \sqrt{\left[\sum_{j \neq i} b_{ij} d_j^*\right]} - 1 \qquad \text{..................................(B)}$$

Since it is not possible to solve this set of equations analytically, we use an iterative learning model to solve these equations.

Let us consider the interaction of users in a real P2P system. Any particular peer Pi interacts only with a limited set of all possible peers — these are the peers who serve files of interest to Pi. As it interacts with these peers, Pi learns of the contributions made by them and to maximize its utility adjusts its own contribution. Obviously this contribution that Pi makes is not globally optimal because it

is based only on information from a limited set of peers. But after Pi has set its own contributions, this information will be propagated to the peers it interacts with and those peers will adjust their own contribution. In this way the actions of any peer Pi will eventually reach all possible

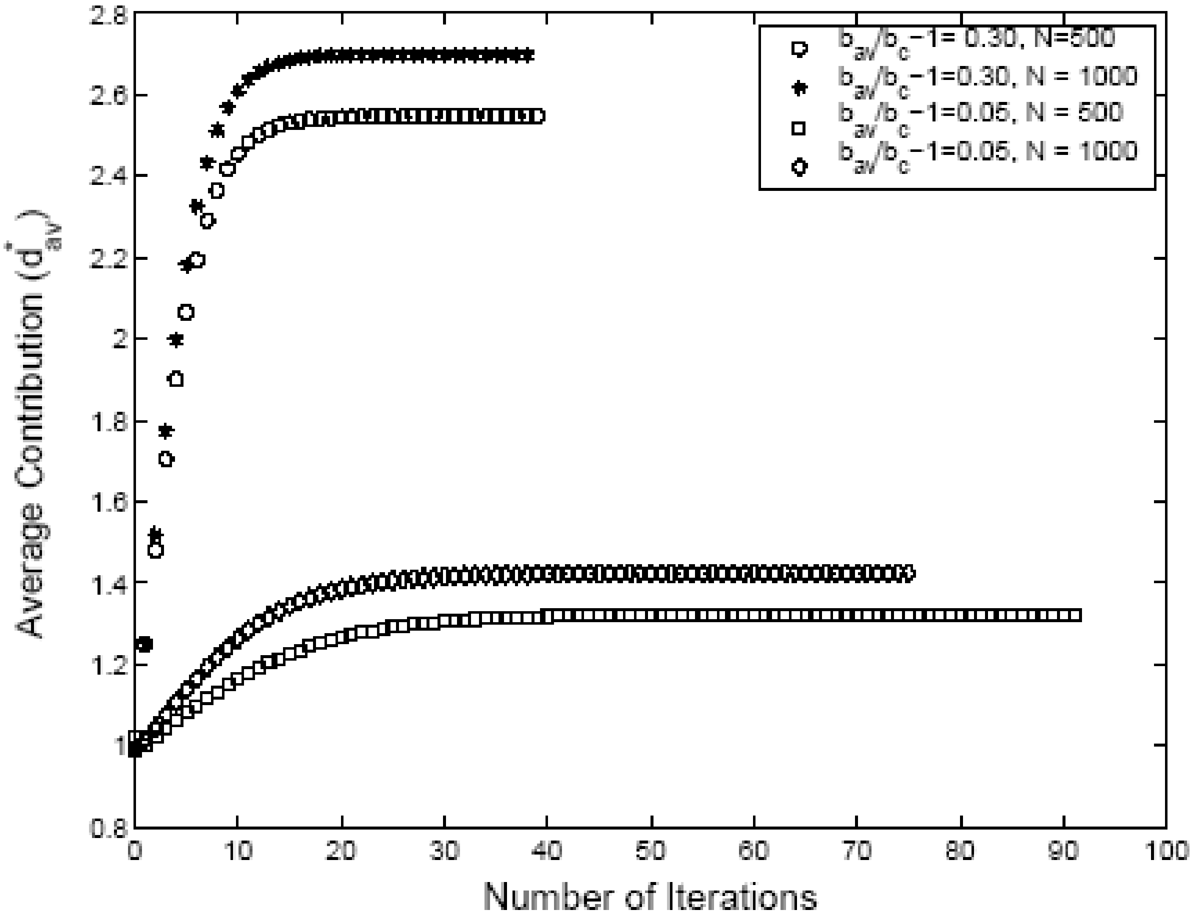
peers. The reaction of the peers to Pi's contribution will affect Pi itself and it will find that perhaps it will be better off by adjusting its contribution once more. In this way, every peer will go through an iterative process of setting its contribution. If and when this process converges, the resulting contributions will constitute a Nash equilibrium.

Algorithms: iterative learning model

1. *di = random contribution*
2. *While (converge == false){*
3. *new_di = computeContribution (d, b);*
4. *if (new_di == di) {*
5. *converge = true;*
6. *}*
7. *di = new_di;*
8. *}*

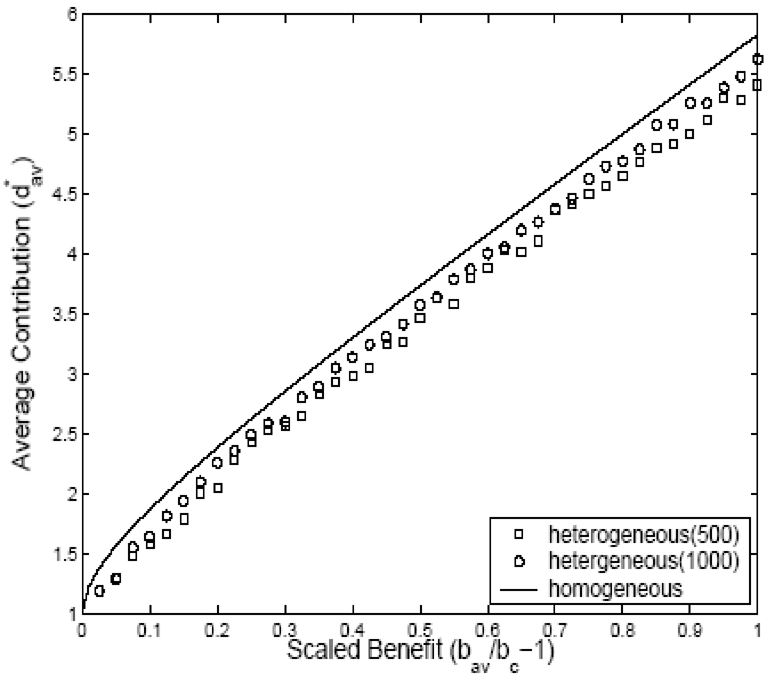
The iterative learning algorithm that we have chosen to solve equation (B) mimics this learning process. To start with, all the peers have some random set of contributions. In a single iteration of the algorithm, every peer $P_i$ determines the optimal value of $d_i$ that it should contribute given the values of d for other peers and the values of $b_{ij}$ . At the end of the iteration the peers update their contribution to their new optimal values. Since now the contributions di are all different, the peers need to recompute their optimal values of $d_i$ and we can start the next iteration. When this iterative process converges to a stable point, we reach a Nash equilibrium. In the following numerical experiments we demonstrate that for heterogeneous system of peers, the iterative learning process does converge to the desirable Nash equilibrium dhi and we compare the results with the analytic results for the system of homogeneous peers.

**Convergence of learning algorithms**



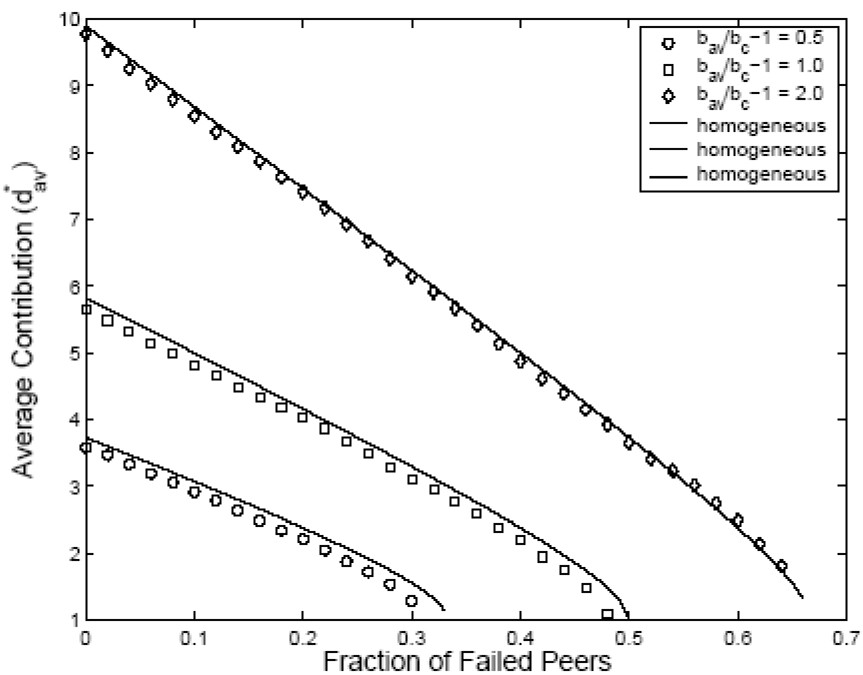

The two data sets correspond to different values of average benefit. Higher the average benefit, faster is the convergence process to equilibrium. As the value of $b_{av}$ approach the critical value $b_c$, approach to equilibrium becomes slower and slower.

## Simulation: $d_{av}$ vs. ($b_{av}/b_c$-1




Above figure shows the equilibrium average contribution by the peers as a function of average benefit. The solid line is the solution from the homogeneous system. As expected, the equilibrium contribution increases monotonically with increasing benefit. For average benefit $b_{av} < b_c$, the iterative algorithm converges to $d_i = 0$. Note that the two sets of results for 500 and 1000 peers almost coincide with each other. So the results are essentially independent of system size.

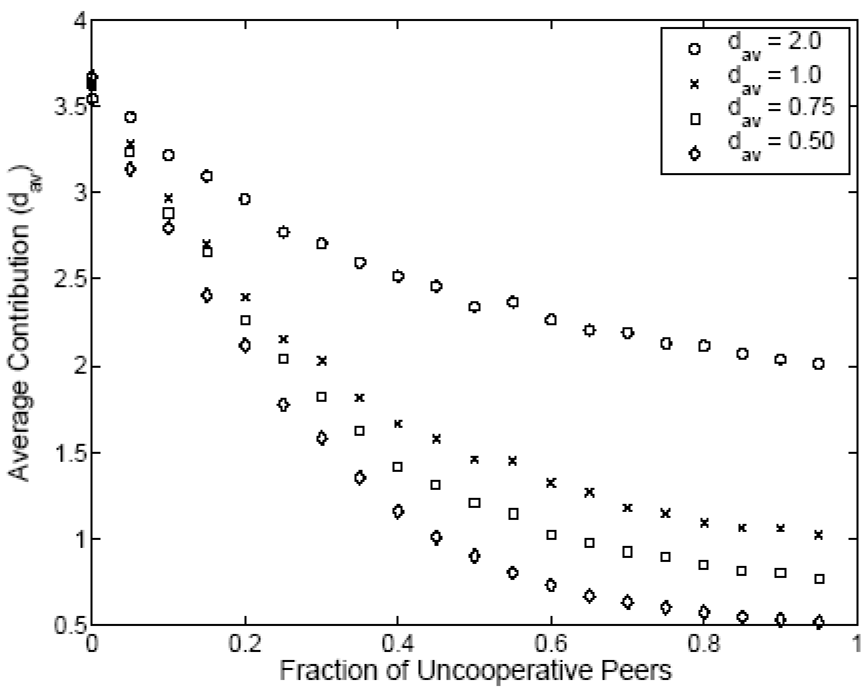
## Simulation: leave system




Above fig. shows the effect of some peers leaving the system. If some peers leave the system, the benefit per peer would be reduced. As the fraction of active peers decrease, the contribution from each of the peers decrease and at some point, the benefits are too low for the peers and the whole system collapses. The system can be pretty robust for high benefits : for a benefit level of (bav − bc)/bc = 2.0, the system can survive until 2/3 of the peers leave the system. In contrast to traditionally fragile distributed systems, we see that for P2P systems robustness increase with size : as the system grows bigger and bigger, benefits for each peer increases and the system becomes more robust to random fluctuations.

<u>**Summary**</u>

- Differential service based incentive model for p2p system that eliminating free riding and increasing availability of the system
- If the benefit $b_i$ is larger than a critical benefit $b_c$, then the peer's best option is to join the system and operate at the Nash equilibrium value of contribution. If on the other hand $b_i < b_c$, the peer is better off not joining the system. When $b_i = b_c$, the peer is indifferent between these two options. These properties are robust and do not depend on the details of the particular incentive mechanism that is used.




Average contribution at Nash equilibrium plotted against fraction of uncooperative peers. Total number of peers is 1000. The labels specify the average contribution of uncooperative peers. Average benefit is $b_{av}/b_c - 1 = 0.5$.