# LECTURE 30

*PARTIAL ORDER*

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## <u>DEFINITION</u> :

A relation R on a set A is called partial order if R is **reflexive ,antistmmetric and transitive** . The set A together with relation R is called **partially ordered set** . Hence Partilly ordered set or POSET is denoted by (A,R) .

**Example :** Let Z+ be a set of positive integers .Then the ususl relation <=(Less than or equal to) is a partial order on Z+ .

The relation < on Z+ is not partial order , since it is not reflexive .

Let R be a partial order on a set A ,and R' be the inverse relation of R. Then R' is also partial order . The poset (A, R') is called the **dual** of the poset (A, R') and R' is called the **dual** of the the partial order R .

**Symbol :** Generally the symbol used to denote partil order is (A, < = ) . But it does not mean that the relation is really " less than or equal to " , here it denotes a general relation.

## Linear Order :

If (A, <= ) is a POSET , a and b are two elements of the poset then a and b are said to be comparable if : a<=b or b<=a .

If every pair of elements is a poset A is **comparable** then A is called Linearly ordered set and the partial order is called linear order .

**THEOREM** : If (A,<= ) and (B,<=) are posets then (A X B ,<=) is a poset with partial order <= defined by :

(a,b) <= (a' ,b') if a<=a' and b <=b' .

**Proof** : If (a,b) belongs to A X B then (a,b) <= (a,b) ,since a<=a' in A , and b <= b' in B.

Hence , it satisfies **Reflexive property** .

Let (a,b)<=(a',b') and (a',b')<=(a,b) .Hence

a<=a' and b<=b'

a'<=a and b'<=b

Hence a = a' and b = b' ,

Therefore it is **antisymmetric** .

Finally , let (a,b)<=(a',b') and (a',b')<=(a",b") .

a<=a' a'<=a"

b<=b' b'<=b"

We get, By transitivity of partial order property on A

a <= a" .

Similarly , b <= b" .

Hence (a,b)<=(a",b") , which proves **transitivity**.

So, (A X B) is a poset .

The partial order defined on A X B is called ***product partial order*** .

Another important partial order on A X B is defined by :

$(a,b)<(a',b')$ if (i) $a<a'$ or if (ii) $a=a'$ and $b<=b'$ .

This ordering is called ***lexicographic ordering*** , or ***dictionary order*** .