Mathematical Induction

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Intoduction :

Mathematical induction is a method of mathematical proof typically used to establish that a given statement is true of all natural numbers. It is done by proving that the first statement in the infinite sequence of statements is true, and then proving that if any one statement in the infinite sequence of statements is true, then so is the next one.

Amongthe various methods of proof,it is a type of proof that deserves special attention is mathematical induction.

However, it must be kept in mind that mathematical induction should not be misconstrued as a form of inductive reasoning, which is considered non-rigorous in mathematics. In fact, mathematical induction is a form of deductive reasoning and is fully rigorous.

Need of induction :

In the proofs given in the geometric series topic there are statements like 'we can continue in this way 'and 'we can generalize for all n req . Such statements are okay for informal proofs. However, for formal proofs something more is needed. For example, suppose I used the following argument to show that 120 is the largest number: 'Since 120 is divisible by 1, 2, 3, 4, 5 and 6 we can continue in this way to show that it is divisible by all numbers'. We need a way to separate such nonsense from valid arguments by providing a way of showing how the proof for each value of n can be derived from the previous value of n. What will be shown is that the method for stating such proofs turns out to be a powerful tool for generating proofs.

We can establish the truth of a proposition if we can show that it follows from smaller instances of the same proposition, as long as we can establish the truth of the smallest instance (or instances) explicitly.

Hence we see that mathematical induction is a very powerful tool for creating proofs.

Strategy during the proof :

The simplest and most common form of mathematical induction proves that a statement involving a natural number n holds for all values of n. The proof consists of two steps

- The basis (base case): showing that the statement holds when n = 0.
- The inductive step: showing that if the statement holds for some n, then the statement also holds when n + 1 is substituted for n.This step is known as the induction step.

The assumption in the inductive step that the statement holds for some n is called the induction hypothesis (or inductive hypothesis). To perform the inductive step, one assumes the induction hypothesis and then uses this assumption to prove the statement for n + 1.

Example :

Mathematical induction can be used to prove that the statement:
1+2+3+.......+n=n(n+1)/2

holds for all natural numbers n. It gives a formula for the sum of the natural numbers less than or equal to number n. The proof that the statement is true for all natural numbers n proceeds as follows.

Call this statement P(n).

Basis

Show that the statement holds for n = 0. P(0) amounts to the statement: 0=0; which is true of course.

In the left-hand side of the equation, the only term is 0, and so the left-hand side is simply equal to 0. In the right-hand side of the equation,

0(0 + 1)/2 = 0.

The two sides are equal, so the statement is true for n = 0. Thus it has been shown that P(0) holds.

Induction step

Show that if P(n) holds, then also P(n + 1) holds. This can be done as follows.

Assume P(n) holds (for some unspecified value of n). It must be shown that then P(n + 1) holds, that is:

1+2+.....+(n+1)=(n+1)(n+2)/2

using that the relation is true for n, we write the equation as

(1+2+......+n)+n+1

=n(n+1)/2 +n+1

=(n+1)(n+2)/2;

hence proved.

Strong Induction :

Another generalization, called strong induction (or complete induction or course of values induction), says that in the second step we may assume not only that the statement holds for n = m but also that it is true for n less than or equal to m.

In other words,if

p(0) is true and

p(1) is true and .

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p(n) is true ,

We will have to show that p(n+1)holds in the induction step.

We can safely say that strong induction differs from the induction only in the induction hypthesis step.